

*Quality growth: from process to product  
innovation along the path of development*

**Esteban Jaimovich**

**Economic Theory**

ISSN 0938-2259

Econ Theory

DOI 10.1007/s00199-020-01266-0



**Your article is protected by copyright and all rights are held exclusively by Springer-Verlag GmbH Germany, part of Springer Nature. This e-offprint is for personal use only and shall not be self-archived in electronic repositories. If you wish to self-archive your article, please use the accepted manuscript version for posting on your own website. You may further deposit the accepted manuscript version in any repository, provided it is only made publicly available 12 months after official publication or later and provided acknowledgement is given to the original source of publication and a link is inserted to the published article on Springer's website. The link must be accompanied by the following text: "The final publication is available at [link.springer.com](http://link.springer.com)".**



# Quality growth: from process to product innovation along the path of development

Esteban Jaimovich<sup>1</sup>

Received: 12 December 2018 / Accepted: 8 April 2020  
© Springer-Verlag GmbH Germany, part of Springer Nature 2020

## Abstract

We propose a demand-driven growth theory where process innovations and product innovations fulfill sequential roles along the growth path. Process innovations must initially set the economy on a positive growth path. However, process innovations alone cannot fuel growth forever, as their benefits display an inherent tendency to wane. Product innovations are therefore also needed for the economy to keep growing in the long run. When the economy fails to switch from a growth regime steered by process innovation to one driven by product innovation, R&D effort and growth will eventually come to a halt. However, when the switch to a product innovation growth regime does take place, a virtuous circle gets ignited. This happens because product innovation effort not only keeps growth alive when incentives to undertake process innovation diminish, but it also regenerates profit prospects from further process innovation effort.

**Keywords** Endogenous growth · Process and product innovation · Non-homothetic preferences · Quality ladders

**JEL Classification** O30 · O31 · O41

---

I would like to thank Pantelis Kazakis, Vincenzo Merella, Ignacio Monzon, Manuel Oechslin, and Gonzague Vannoorenberghé for comments and suggestions, as well as seminar participants at the Collegio Carlo Alberto, University of Surrey, the Development Economics Workshop at Tilburg University, the Royal Economic Society Meeting at University of Bristol, the LACEA Meeting 2017, and the RIDGE Macroeconomics and Development Meeting 2018.

---

**Electronic supplementary material** The online version of this article (<https://doi.org/10.1007/s00199-020-01266-0>) contains supplementary material, which is available to authorized users.

---

✉ Esteban Jaimovich  
e.jaimovich@surrey.ac.uk

<sup>1</sup> School of Economics, University of Surrey, Guildford GU2 7XH, UK

## 1 Introduction

Process and product innovations are two key determinants behind sustained economic growth. Process innovations introduce technological improvements that allow an expansion in the quantity of goods that an economy can deliver. Product innovations foster growth by bringing to the market goods of higher quality than those previously available. This paper presents a demand-driven growth theory in which both types of innovations fulfill crucial roles, and where their respective roles display a specific sequential timing. Our theory shows that process innovations must precede product innovations along the path of development. Yet, while process innovations can initially set the economy on a positive growth path, they cannot sustain rising incomes perpetually. Long-lasting growth requires that the economy is also able to start generating product innovations at some point. The reason for this is that, without the help of quality-upgrading innovations, the incentives to invest in process innovations will eventually start to wane as physical production increasingly expands.

The model features an economy with a vertically differentiated good, potentially available in an infinite number of (vertically ordered) quality levels. All the quality levels are produced with technologies that use labor as their sole input. Both labor productivity and the degree of vertical differentiation are endogenous to the model. Labor productivity increases as a result of process innovations. In particular, process innovations lower the costs of production (in terms of hours of labor), leading to an increase in the physical quantities that may be produced with a given amount of labor. Product innovations instead allow the introduction of better quality versions of the vertically differentiated good.

Innovations are the outcome of purposeful research and development effort. Hence, investment in process and in product innovations will be the endogenous response to the potential profit associated with each of them. The underlying force leading to their different timings along the growth path stems from our demand side. Individuals exhibit non-homothetic preferences along the quality dimension. In particular, their willingness to pay for varieties of higher quality increases as their incomes rise. An implication of this is that product innovations tend to become increasingly profitable along the growth path, since product innovators can charge higher markups when they face richer consumers. However, our non-homothetic demand structure entails also a flip side. At early stages of development, the economy must rely on process innovations as the source of income growth. This is because the low willingness to pay for quality by consumers with low incomes stifles profit opportunities for product innovators.

Our theory then shows that at early stages of development process innovation must become the leading actor. Product innovation takes over a more prominent role instead in mature economies. Furthermore, such transition from process to product innovation proves essential for sustaining growth in our model. In a context where individuals display decreasing marginal utility on physical consumption, process innovations bring about two opposing dynamic forces. On the one hand, they drive the marginal utility of consumption down. Thus, the

prospects of *future* profits from process innovation are endogenously dampened by *current* process innovation efforts. On the other hand, the higher quantity of consumption allowed by process innovations is exactly what spurs profit prospects from product innovation effort. The tension between these two countervailing forces means that an economy may or may not succeed in eventually switching from a growth path steered by process innovation to one steered by product innovation. When it fails to do so, growth will eventually come to a halt due to the negative effect of decreasing marginal utility on process innovation profits.

The switch to a product innovation growth regime can in turn ignite a virtuous circle with further process innovation down the road. The incapacity of cost-cutting innovations to spur growth perpetually lies in that a continuous expansion in quantity of production must struggle against the decreasing marginal utility of (physical) consumption that it simultaneously leads to. This struggle makes it increasingly hard to keep profit prospects from process innovation high enough to sustain it forever. Product innovations are able to relax this inherent tension. In particular, by raising the intrinsic quality of goods offered in the economy, product innovations make the decreasing marginal utility of (physical) consumption less pressing, and thereby regenerate profit prospects from further process innovation effort.

Our theory yields thus a model where long-run growth stems from a positive feedback loop between cost-cutting and quality-enhancing innovations. The endogenous growth literature has produced several models where growth results from the interplay of different types of innovation effort, such as general purpose technologies and sector-specific complementary inputs [e.g., Bresnahan and Trajtenberg (1995), Helpman and Trajtenberg (1998)], fundamental research/breakthroughs and secondary development [e.g., Jovanovic and Rob (1990), Aghion and Howitt (1996), Redding (2002)], and invention and learning-by-doing [e.g., Young (1993), Stein (1997)]. All these models share a common trait: There exists one fundamental source of long-run growth that interacts with a transitional and bounded source of growth linked to the fundamental technology. Our model departs from the notion of fundamental and secondary sources of technical change. We look at the specific case of process and product innovations as *potentially* independent sources of technical change, but *ultimately* unable to sustain unbounded growth without one another.

The number of articles dealing separately with either process or product innovation effort in the endogenous growth literature is huge. Yet, it is hard to find models where both are explicitly involved together in steering the economy along the growth path, while playing a distinctive role as growth engines.<sup>1</sup> One prominent

<sup>1</sup> Models where growth is the result of the new technologies that allow an increase in physical production (i.e., process innovations) can be found in: Shleifer (1986), Aghion and Howitt (1992), Jones (1995), and Kortum (1997). Examples of models where growth is driven by the introduction of final goods of higher quality than before (i.e., product innovations) are: Segerstrom et al. (1990), Grossman and Helpman (1991a, b), Stokey (1991), and Segerstrom (1998). A third type of innovation, which is neglected by our model, is that one that leads to a horizontal expansion in the variety of goods, as in Judd (1985), Romer (1990), Grossman and Helpman (1991c, Ch. 3), Young (1993). We relegate to the concluding section a brief discussion on the possible effects of introducing variety-expanding innovations within the context of our model.

example is Foellmi et al. (2014), who build a growth model with non-homothetic preferences where firms must choose between product innovations to introduce new luxury goods to be consumed only by the rich, or process innovations that turn luxuries into mass consumption goods also available to the poor. Their model depicts situations where this type of product cycle arises as the optimal behavior by firms, and uses it to explain how some new goods first introduced during the 20th century have later on become available as mass consumption goods (e.g., automobiles, refrigerators, etc.).<sup>2</sup> The main focus of our model is somewhat different, as it studies how the interplay between process and product innovation efforts can sustain a continuous increase in incomes in the long run and how the preeminence of each type of innovation shifts along the growth path. In that sense, our model is mostly concerned with how an economy may keep growing *beyond* a mass consumption economy, in a context where rising incomes increasingly tilt consumer preferences toward quality expansion (and away from quantity expansion).<sup>3</sup>

Besides Foellmi et al. (2014), a few other articles have mingled together vertical and horizontal innovations; see, e.g., Peretto (2007), Peretto and Connolly (2007), Sorger (2011), Chu et al. (2012), Akcigit and Kerr (2018). These articles, however, have all remained within standard homothetic frameworks, where trade-offs and interactions faced by innovators when choosing between process and product innovations are disconnected from changes of consumer behavior along the growth path. As a result, these models remain silent about the needed transition from process to product innovation in order to keep richer consumers' demands continuously unsatiated.

A key aspect behind our demand-driven growth model is therefore the non-homotheticity of preferences along the quality dimension (i.e., the notion that willingness to pay for quality upgrading rises with income). This is in fact a property of the preference structure that has been previously incorporated in several trade models [e.g., Flam and Helpman (1987), Murphy and Shleifer (1997), Fajgelbaum et al. (2011, 2015), Jaimovich and Merella (2012, 2015)], and it has also been widely supported by both household-level data based on consumer surveys [e.g., Bils and Klenow (2001)] and bilateral trade flows data [e.g., Hallak (2006) and Choi et al. (2009)]. In our model, quality upgrading arises endogenously as the result of firms' effort to cater to consumers with rising incomes.<sup>4</sup> In addition to

---

<sup>2</sup> Matsuyama (2002) also studies an endogenous growth model where goods initially affordable to the rich become gradually mass consumption goods affordable to all individuals. In his model, however, technological change is not the result of purposeful R&D effort, but it arises because of industry-specific learning-by-doing.

<sup>3</sup> Foellmi and Zweimüller (2006) also present a demand-driven endogenous growth model where individuals display non-homothetic preferences. Their model differs from ours substantially, with two key differences: (i) In their model, there is no quality differentiation (their non-homotheticities are the result of hierarchical preferences with a horizontal continuum of goods); (ii) their model features only cost-cutting innovations, which is combined with a setup cost that must be incurred to open new sectors/product lines.

<sup>4</sup> This is also a phenomenon that has received support by a growing strand of empirical papers; e.g. Verhoogen (2008), Brambilla et al. (2012), Manova and Zhang (2012), Bas and Strauss-Kahn (2015), and Flach (2016).

such quality-upgrading effort in response to rising incomes, our theory also predicts the need of a gradual shift from process innovation to product innovation along the growth path.

The need to transition from a growth regime heavily grounded on cost-reducing innovations toward one more strongly reliant on quality-enhancing innovations is a notion that in fact resonates well with current policy discussions in economies facing up to the so-called middle-income trap. The most paradigmatic example is probably China, and its response to it is encapsulated in the strategic plan dubbed 'Made in China 2025.' This is a broad long-run growth platform, but primarily aimed at bolstering the quality of production in the Chinese manufacturing sector, and explicitly emphasizing 'quality over quantity.' In fact, the deep concern regarding quality of Chinese products has been openly voiced by China's Premier Li Keqiang, stating that a key challenge ahead for China is '[to] redouble its efforts to upgrade from a manufacturer of quantity to one of quality.'<sup>5</sup> Quite remarkably, and acknowledging the relevance of non-homotheticities along the quality dimension of consumption, one of the stated rationales behind the 'Made in China 2025' initiative is 'to be able to meet the higher demand for quality and high-end products by the growing middle-income Chinese population.'

The risks faced by the Chinese economy have also been recently raised more formally by Zilibotti (2017), warning that a shift to innovation is a necessary condition for keeping Chinese growth at a similar pace as in recent years. Wei et al. (2017) expand on this issue by highlighting that, given the rising labor costs faced by Chinese firms, China must start focusing on quality upgrading, so that firms will not need to depend on production of low-cost goods anymore. One key aspect of our theory is that growing incomes not only eventually call for product innovations to take place, but that they also help spur the incentives to introduce newer and better goods, so as to substitute lower-quality versions of them. Recent evidence from Chinese consumers with rising incomes in Beerli et al. (2020) also grants support to this point, showing the case of progressive saturation of the market for low-quality transportation vehicles and its substitution by higher-ranked ones.<sup>6</sup>

Besides the explicit acknowledgement in the policy arena of eventual growth saturation in the absence of quality-improving innovations, recent studies relying on firm-level data also exhibit systematic support for this notion. For example, Argente et al. (2019), using 12-digit product barcode data in the USA, find that variation of product quality explains nearly 90% of the time-series dynamics in sales of consumer products over firms' life cycle. On the other hand, they show that cutting

<sup>5</sup> See reference in [http://www.china.org.cn/business/2015-03/30/content\\_35192417.htm](http://www.china.org.cn/business/2015-03/30/content_35192417.htm).

<sup>6</sup> The idea of output quality-enhancing as a crucial factor to keep growth alive when confronting consumers with rising incomes was acknowledged as well by post-war Japan. This led to the Japanese productive reorganization strategy known as 'The Quality Revolution.' The main idea behind this strategy was to undertake a long sequence of drastic improvements in quality, with the ultimate aim to expand Japanese exports to richer consumers in the USA and Europe. By the early 1980s, the Japanese strategy had proved so successful that American firms had embark themselves in their own 'quality revolution,' as a way to restore their domestic market shares which had been grossly hit by high-quality manufacturing imports from Japan (Kolb and Hoover 2012).



prices via cost-cutting innovations play only a secondary and minor role. In particular, they conclude that demand factors driven by preferences for improved varieties of consumer goods are the main factor behind the life-cycle patterns of firms' growth in the USA.<sup>7</sup> Also based on US barcode data, Hottman et al. (2016) show that between 50 and 75% of variation in firms sales in the USA is explained by differences in the overall quality/appeal of the goods they offer. Similar results are also obtained by Garcia-Macia et al. (2019), using in this case US census data on firms' labor force growth: They find that over 60% of US firms' growth (measured by their labor force) is the result of quality improvements of the set of goods they produce. Firm-level data therefore consistently point out that, in developed economies like the USA, quality-upgrading innovations are the key factor for firms to be able to remain on a positive growth path.<sup>8</sup>

The above empirical studies have managed to separate the contribution to firms' growth of cost-cutting innovations versus quality-improving innovations, concluding that in developed economies it is the latter that takes the lion's share. Yet, when considering growth at the aggregate level, it is hard to argue that process and product innovations do not interact with each other and also reinforce one another. In fact, the overall intensity of R&D investment is found to be systematically higher in richer economies than in poorer ones, and this is the case for both process and product innovations. The model we present aims at providing a rationale for why cost-cutting and quality-upgrading innovations are *both* crucial for sustaining long-run growth, and feedback on each other along the growth path. At the same time, the model showcases a growth path where quality-upgrading innovations are particularly needed in order to set economies on a growth path that avoids coming to halt in a sort of 'medium-income trap.'<sup>9</sup> Lastly, the model features a growth path where product innovations tend to gradually gain prevalence as the main growth engine of mature economies.

The rest of the paper is organized as follows. Section 2 presents the setup and main assumptions of the model. Section 3 studies a simplified framework where the only source of technical change is process innovations, showing that growth is eventually bound to come to a halt. Section 4 introduces product innovations and shows

---

<sup>7</sup> Related evidence based on barcode data is also presented by Argente et al. (2018) who, exploiting the variation of GDP per head over the period 2007–2013, show that the intensity of product innovation by US firms has been strongly procyclical.

<sup>8</sup> Although there exists an empirical literature that investigates firms' R&D investment differentiating between process and product innovation [e.g., Cohen and Klepper (1996), Huergo and Jaumandreu (2004), Parisi et al. (2006), Harrison et al. (2014), Peters et al. (2017)], none of these papers uses data collected from a large and diverse sample of countries. These studies either use firm-level data from one single country (like Cohen and Klepper (1996) for USA, Huergo and Jaumandreu (2004) for Spain, Parisi et al. (2006) for Italy, or Peters et al. (2017) for Germany), or from a small number of countries with similar levels of income (like Harrison et al. (2014) for France, Germany, Spain, and UK). As a result, this literature is silent about correlations between income per capita and intensity of investment in process innovation relative to product innovation.

<sup>9</sup> See Marsiglio and Tolotti (2018) for a growth model featuring middle-income traps as a result of coordination failures in the presence of social interactions.



that they may help sustaining growth for longer. Section 5 describes the main features of the long-run dynamics of the model. Section 6 provides some concluding remarks. All relevant proofs are relegated to Appendices.

## 2 Setup of the model

Life evolves in discrete time over an infinite horizon  $\mathcal{T} = \{1, 2, \dots, \infty\}$ . The economy is populated by a non-overlapping sequence of households. Each household is alive only for one period. In each period of time along  $\mathcal{T}$ , only one single household is alive. Once a household ceases to exist, it is immediately superseded by a newly born household.

All households consist of two members:  $W$  and  $R$ . Household members denoted by  $W$  are endowed with one unit of time, which they supply inelastically to firms as labor. Those denoted by  $R$  are also endowed with one unit of time, but they can choose either to enjoy it as leisure time or, alternatively, to supply it as R&D effort to firms. We will often refer to  $W$  as *workers* and to  $R$  as *researchers*. Also, we will often index household members by the period  $t \in \mathcal{T}$  during which they are alive; that is,  $W_t$  and  $R_t$  will denote, respectively, the worker and the researcher alive in  $t$ .

In addition to the households, the economy comprises also a non-overlapping sequence firms. Like households, firms are only active for one period of time, after which they are replaced by a new generation of firms. Setting up a firm entails no cost. Firms are risk neutral. Any profit made by a firm active in  $t$  is distributed to the household alive in  $t$ .

### 2.1 Technologies and goods

The economy's output consists of a final consumption good produced by firms. The consumption good is *potentially* available in an unbounded number of vertically ordered quality levels:  $q \in Q$ . We normalize the lowest value of  $Q$  to unity. We will recurrently refer to the different quality levels as *quality versions*, and to the lowest quality ( $q = 1$ ) as the *baseline quality*.

#### 2.1.1 Prehistoric technology

At the beginning of the time horizon ( $t = 1$ ), the economy inherits a technology from the prehistoric period  $t = 0$ . The *prehistoric technology* allows transforming one unit of labor time into one unit of consumption good, but only in its baseline quality version  $q = 1$ . All firms active in  $t = 1$  have free access to this technology.

In addition to having access to the prehistoric technology, firms may hire researchers to undertake R&D effort and create new technologies. Innovations can be of two different types: i) *process innovations*, which raise labor productivity, but do not increase the quality of the consumption good, and ii) *Product innovations*, which lead to the introduction of higher-quality versions of the consumption good,

but leave labor productivity unaffected. We describe both types of innovations in greater detail in the next two subsections.

We assume that firms can innovate at most once during their lifetimes. When a firm generates an innovation, the firm is granted a patent on it. Once the firm ceases to exist, the patent expires and all the know-how contained in it becomes freely available to all other future generation of firms. We broadly refer to all the technologies that result from a defunct patent as *inherited technologies*. Inherited technologies may be further improved upon through *current* R&D effort (either in the form of process or product innovation effort).

### 2.1.2 New and inherited technologies I: the effects of process innovation

Consider a generic firm active in period  $t$  that hires the researcher  $R_t$  to undertake *process* innovation effort. Each unit of process innovation effort exerted by  $R_t$  allows the firm that hired the researcher to increase labor productivity by  $\sigma$  units above inherited technology's labor productivity.

In order to keep track of how technologies evolve over time, it proves convenient to define an indicator function  $\mathbf{I}_p(t)$ , which equals 1 when  $R_t$  is hired by a firm to undertake *process* innovation effort, and 0 otherwise. By using  $\mathbf{I}_p(t)$ , we can next define

$$A_{t-1} \equiv \sum_0^{t-1} \mathbf{I}_p(\tau), \tag{1}$$

which denotes the (historical) number of researchers that have been hired by some firm to undertake process innovation effort before period  $t$ . Based on (1), we can now describe formally the set of technologies available to firms active in  $t$  resulting from process innovation effort.

**Assumption 1** Consider some generic period  $t \in \mathcal{T}$ , in which one of the active firms hires the researcher  $R_t$  to exert process innovation effort. The technologies available for producing the baseline quality variety (i.e.,  $q = 1$ ) in period  $t$  are given by:

(i) **Inherited technology:** Any firm active in  $t$  will be able to produce

$$1 + \sigma A_{t-1}$$

units of  $q = 1$  with one unit of labor, where  $\sigma > 0$  and  $A_{t-1}$  is given by (1).

(ii) **New technologies:** The firm that hired the researcher  $R_t$  will be able to produce

$$1 + \sigma A_{t-1} + \sigma$$

units of  $q = 1$  with one unit of labor, where  $\sigma > 0$  and  $A_{t-1}$  is given by (1).

The first part of Assumption 1 stipulates that the effects of process innovation on labor productivity accumulate over time and are (freely) transferred to *future* generation of firms as inherited technologies once the current innovator disappears. The second part describes the effects of *current* process innovation: A process

innovation generated in period  $t$  increases labor productivity by  $\sigma > 0$  units relative to the inherited technology received in  $t$ .

### 2.1.3 New and inherited technologies II: the effects of product innovation

Improved technologies may also originate from product innovation effort. Consider now a generic firm active in period  $t$  that hires  $R_t$  to undertake *product* innovation effort. Each unit of product innovation effort exerted by  $R_t$  allows the firm that hired the researcher to introduce an improved version of the consumption good that is  $\rho > 0$  quality units above the quality version available before period  $t$ .

Analogously as done for process innovations, it proves convenient to define now an indicator function for the historical levels of product innovation effort, namely  $\mathbf{I}_q(t)$ , which equals 1 when  $R_t$  is hired by a firm to undertake *product* innovation effort, and 0 otherwise. By means of  $\mathbf{I}_q(t)$ , we can next also define

$$q_t \equiv 1 + \rho \sum_0^t \mathbf{I}_q(\tau), \tag{2}$$

where  $q_t$  will be equal to highest quality version of the good available in period  $t$ .

**Assumption 2** Consider some generic period  $t \in \mathcal{T}$ , in which one of the active firms hires the researcher  $R_t$  to exert product innovation effort. Then:

(i) **Inherited technology:** Any firm active in  $t$  will be able to produce

$$(1 + \sigma A_{t-1})/q_{t-1} \tag{3}$$

units of the consumption good in the quality level  $q_{t-1}$ , where  $q_{t-1} = 1$  if  $t = 1$  and  $q_t \equiv 1 + \rho \sum_0^{t-1} \mathbf{I}_q(t)$  for  $t \geq 2$ .

(ii) **New technologies:** The innovative firm in  $t$  will be able to produce

$$(1 + \sigma A_{t-1})/q_t \tag{4}$$

units of the consumption good in the quality level  $q_t$ , where  $q_t = q_{t-1} + \rho$ .

Assumption 2 describes how a successful product innovation generated in period  $t$  allows the production of a version of the good whose quality level is  $\rho > 0$  units above that one available before  $t$ . Two additional remarks about Assumption 2 are worth stressing here. Firstly, Assumptions 1 and 2 taken together imply that past process innovations generate productivity improvements that are *not* quality specific. More precisely, the numerators in (3) and (4) entail that improvements in labor productivity owing to *prior* process innovations apply identically to *all* the existing quality versions of the consumption good.<sup>10</sup> Secondly, the denominators in (3) and

<sup>10</sup> While the assumption that prior process innovations apply *identically* to all quality versions of the good may seem extreme, all our main results would still hold when process innovations spillovers applied only *partially* to higher-quality versions of the good, similarly to Jovanovic and Nyarko (1996) or Redding (2002).

(4) entail that the unit labor requirements are greater for higher-quality versions of the consumption good.

## 2.2 Household preferences

A household alive in period  $t$  is characterized by the following utility function:

$$U_t = \ln \left[ \sum_{q \in Q_t} [q x_t(q)]^q \right] + \eta(1 - \varepsilon_t). \tag{5}$$

In (5),  $x_t(q) \in \mathbb{R}_+$  denotes the quantity of the quality version  $q \in Q_t$  consumed by the household alive in period  $t$  and  $Q_t \subseteq Q$  is the set of quality versions available in  $t$ . The variable  $\varepsilon_t \in \{0, 1\}$  takes the value of 1 if  $R_t$  decided to sell his time endowment as R&D effort to a firm (either in the form of process or product innovation), and 0 if he instead chose to use it as leisure, with  $\eta > 0$  being the utility of leisure.<sup>11</sup>

Two important properties of (5) are worth stressing. Firstly, since the lowest value of the set  $Q_t$  is  $q = 1$ , the term  $\sum_{q \in Q_t} [q x(q)]^q$  turns out to be a sum of convex functions in  $x(q)$ . As a consequence of this, in the optimum, households will select a *corner* solution for their consumption plan, that is, a solution characterized by  $x(q) > 0$  for some  $q \in Q_t$  and zero for all other quality versions. Secondly, the expression  $[q x(q)]^q$  in (5) means that higher-quality versions magnify the level of utility obtained from a given physical amount of the consumption good. Moreover, this magnifying effect becomes stronger the larger the value of physical consumption  $x(q)$ . This is a crucial feature of our model, as it will lead to a non-homothetic behavior in the demand for quality. In particular, the exponential effect of  $q$  on physical consumption  $x(q)$  leads to demand functions where the willingness to pay for higher versions of the final good is increasing in the level of spending of the household.<sup>12</sup>

## 2.3 Equilibrium in period $t$

In order to define the equilibrium of the economy in period  $t$ , it proves convenient to introduce some further auxiliary notation. We let  $f_{k,t}$  denote a generic firm  $k$  that

<sup>11</sup> The main role played by the utility of leisure parameter  $\eta$  is that of setting a shadow price for R&D effort. In addition, the fact that the leisure utility component enters additively into (5) implies that the R&D effort shadow price is independent of quantity of production and relative market prices. As it will become clear later on in Sects. 3 and 4, a positive value of  $\eta$  opens up the possibility of R&D effort and growth coming to halt along the growth path. On the other hand, the increasing preeminence of product innovation along a positive long-run growth path is independent of the utility of leisure, but actually driven the non-homotheticity of (5) along the quality dimension of consumption.

<sup>12</sup> In growth models with homothetic preferences, the distinction between reductions in costs per physical unit of production (i.e., process innovation) and increases in quality per physical unit (i.e., product innovation) may usually be blurred, since those sources of technical change can often be re-labeled to become isomorphic. Our non-homothetic preference structure in (5) actually turns the distinction between quality improvements and physical productivity improvements economically meaningful, as the income-dependent willingness to pay for quality upgrading implies that those sources of growth cannot be taken simply as isomorphic to one another.

is active in  $t$ , and  $\mathcal{F}_t$  denote the set of active firms in  $t$ . In addition, recalling that  $Q_t$  represents the set of quality versions available in period  $t$ , we let  $Q_{k,t} \subseteq Q_t$  denote the subset of quality versions that firm  $f_{k,t}$  is able to offer during  $t$ .

**Definition 1** (Equilibrium in Period  $t$ ) An equilibrium in period  $t$  features: i) a vector  $(\omega_{k,t}, r_{k,t}, \{p_{k,t}(q)\}_{q \in Q_{k,t}})$  set by each active firm  $f_{k,t} \in \mathcal{F}_t$ , comprising a wage offer  $\omega_{k,t} \in \mathbb{R}_+$ , a researcher remuneration offer  $r_{k,t} \in \mathbb{R}_+$ , and collection of prices  $p_{k,t}(q) \in \mathbb{R}_+$  for each  $q \in Q_{k,t}$ , and ii) a vector  $(\{x_t(q)\}_{q \in Q_t}, \varepsilon_t)$  of household choices, comprising a collection of consumption choices  $x_t(q) \in \mathbb{R}_+$  for each  $q \in Q_t$ , and an R&D effort level  $\varepsilon_t \in \{0, 1\}$  by  $R_t$ , such that:

- a) The worker  $W_t$  sells his time endowment to the firm  $f_{k',t} \in \mathcal{F}_t$  whose  $\omega_{k',t} = \max\{\omega_{k,t}\}_{f_{k,t} \in \mathcal{F}_t}$ .
- b) The researcher  $R_t$  sells his time endowment to the firm  $f_{k'',t} \in \mathcal{F}_t$  whose  $r_{k'',t} = \max\{r_{k,t}\}_{f_{k,t} \in \mathcal{F}_t}$ , provided

$$U_t^*(\varepsilon_t = 1; \omega_{k',t}, r_{k'',t}, \{p_t^{\min}(q)\}_{q \in Q_t}) > U_t^*(\varepsilon_t = 0; \omega_{k',t}, r_{k'',t}, \{p_t^{\min}(q)\}_{q \in Q_t}), \quad (6)$$

while  $R_t$  sets  $\varepsilon_t = 0$  when (6) fails to hold true.

- c) The household alive in  $t$  maximizes (5), given the set of minimum prices  $\{p_t^{\min}(q)\}_{q \in Q_t}$  at which each  $q \in Q_t$  is offered in the market, and given their total earnings  $\omega_{k',t} + \varepsilon_t \times r_{k'',t}$ .
- d) When (6) holds true, firm  $f_{k'',t} \in \mathcal{F}_t$  requests  $R_t$  to exert the type of innovation effort (i.e., process or product) that yields the higher surplus to the firm.
- e) No firm  $f_{k,t} \in \mathcal{F}_t$  would be strictly better off if it offered a vector  $(\tilde{\omega}_{k,t}, \tilde{r}_{k,t}, \{\tilde{p}_{k,t}(q)\}_{q \in \tilde{Q}_{k,t}}) \neq (\omega_{k,t}, r_{k,t}, \{p_{k,t}(q)\}_{q \in Q_{k,t}})$ , given: the technologies available to firm  $f_{k,t}$ , the choices made by all other firms  $f_{j,t} \in \mathcal{F}_t$  with  $j \neq k$ , and the choices made by the household alive in  $t$ .

The equilibrium described above represents a situation in which the household in period  $t$  is a price taker in all markets, and chooses optimally their action given the constellation of prices set by firms in  $\mathcal{F}_t$ . That implies that they will sell their labor time endowment to one of the firms offering the highest wage, and they will only (possibly) buy a given quality version  $q \in Q_t$  from a firm that charges the lowest price for that quality version. In addition, in case the household chooses to sell  $R_t$ 's time endowment (rather than consuming it as leisure), they will sell it as well to one of the firms offering the highest researcher remuneration.

Although not explicitly stated in the equilibrium definition, given that there is free entry and no setup cost for firms, the equilibrium above will always lead to situations satisfying zero-profit conditions by firms (otherwise, some firm in the set  $\mathcal{F}_t$  could have a profitable deviation to exploit, contradicting condition 'e' in the equilibrium definition).

One final aspect to notice from the equilibrium definition is that it postulates that firms are the agents that choose the type of R&D effort researchers must put,

and that only one type of R&D effort can be exerted at a time.<sup>13</sup> In other words, households will only choose whether or not to sell  $R_t$ 's time endowment to firms for a given remuneration, and it is firms that decide next how to use the researcher's time.<sup>14</sup>

### 3 Endogenous growth via process innovation

In this section, we study the dynamic behavior of the economy under the assumption that only process innovations are feasible. Doing this allows a cleaner description of the conditions under which process innovation arises in equilibrium, and how it endogenously generates its own tendency to eventually come to a halt. In what follows, and throughout the whole paper, we take the baseline quality version of the consumption good as the *numeraire*.

Recall from Assumption 1 that all firms active in  $t$  can freely access a technology that allows them to produce  $1 + \sigma A_{t-1}$  units of baseline quality with one unit of labor. Competition among firms for worker  $W_t$  will thus lead to an equilibrium wage

$$\omega_t = 1 + \sigma A_{t-1}. \tag{7}$$

Firms will also compete for the R&D effort that the researcher  $R_t$  can supply to them. Competition by firms for R&D effort time implies that, in equilibrium, the researcher will be offered a total remuneration equal to the surplus it generates. Bearing in mind Assumption 1, it follows that the remuneration offered to  $R_t$  in equilibrium will have to be equal to  $\sigma$  per unit of effort time.<sup>15</sup>

Let now  $Y_t$  denote the lifetime earnings of the household alive at time  $t$ . Using (7), together with the fact that in equilibrium the researcher remuneration for process innovation effort will be equal to  $\sigma$ ,  $Y_t(\varepsilon_t)$  will thus given by:

$$Y_t(\varepsilon_t) = \begin{cases} 1 + \sigma A_{t-1} + \sigma, & \text{if } \varepsilon_t = 1, \\ 1 + \sigma A_{t-1} & \text{if } \varepsilon_t = 0. \end{cases} \tag{8}$$

From (8), the following result obtains quite straightforwardly.

**Lemma 1** *Consider an economy in period  $t$  where the set of active firms  $\mathcal{F}_t$  inherit a technology that allows them to produce  $1 + \sigma A_{t-1}$  units of the baseline quality with one unit of labor. Then, the equilibrium in period  $t$  will feature  $\varepsilon_t^* = 1$  if and only if the following condition holds true:*

<sup>13</sup> Online Appendix C briefly discusses how the model can be extended to allow firms to split  $R_t$ 's time endowment between process innovation effort and product innovation effort.

<sup>14</sup> The equilibrium definition could, in principle, be extended to allow households decide *which* type of R&D effort to exert (rather than leaving that decision to firms), by letting firms offer different remuneration packages according to the type of R&D effort exerted by researchers.

<sup>15</sup> To see this, note that the firm that hired  $R_t$  in order to exert process innovation effort will be able to produce  $1 + \sigma A_{t-1} + \sigma$  units of the baseline quality good by hiring  $W_t$ , to whom it must pay  $1 + \sigma A_{t-1}$  as wage, leaving thus a surplus equal to  $\sigma$  to the firm.

$$\ln \left( 1 + \frac{\sigma}{1 + \sigma A_{t-1}} \right) > \eta. \tag{9}$$

The result in Lemma 1 states that an equilibrium where  $\varepsilon_t^* = 1$  requires that the *additional utility* of consumption allowed by  $Y_t(\varepsilon_t = 1)$ , relative to that obtained from  $Y_t(\varepsilon_t = 0)$ , more than compensates the disutility of effort incurred when setting  $\varepsilon_t = 1$ .

Note from (9) that  $\sigma/(1 + \sigma A_{t-1})$  is strictly decreasing in  $A_{t-1}$ , and it converges to zero when  $A_{t-1} \rightarrow \infty$ . This implies that, for a sufficiently large value of  $A_{t-1}$ , condition (9) will fail to hold true, and period  $t$  will thus feature an equilibrium with  $\varepsilon_t^* = 0$  (i.e., an equilibrium with no process innovation effort). Furthermore, the ratio  $\sigma/(1 + \sigma A_{t-1})$  is strictly increasing in  $\sigma$ . As a result, the threshold value of  $A_{t-1}$  beyond which the equilibrium will fail to exhibit positive effort in process innovation tends to be greater for economies with a larger  $\sigma$ . The exact value of the threshold beyond which (9) fails to hold is pinned down by the level of  $A_{t-1}$  that equalizes the expression in the left-hand side of (9) to  $\eta$ . Namely,

$$\bar{A}(\sigma) \equiv \frac{1}{(e^\eta - 1)} - \frac{1}{\sigma}. \tag{10}$$

Given that we are interested in studying growth events where there is at least one instance with active R&D effort, we impose henceforth the following parametric restriction:

**Assumption 3**  $\sigma \geq e^\eta - 1$ .

Notice that when Assumption 3 fails to hold,  $\bar{A}(\sigma) < 0$ , implying that the economy will *never* experience any process innovation in equilibrium, and the only technology available to firms throughout the entire time horizon  $\mathcal{T}$  will thus be the *pre-historic technology*.

Combining (10) with the result in Lemma 1, we can observe that when Assumption 3 holds true, there is a maximum length of time during which the economy will be able to support an equilibrium with positive process innovation effort. The following proposition presents this argument formally.

**Proposition 1** *Let Assumption 3 hold true, and let  $\bar{t} \equiv \text{integer}\{\bar{A}(\sigma) + 1\}$ , where  $\bar{A}(\sigma)$  is given by (10). Then, the economy will experience an equilibrium with positive process innovation effort from  $t = 1$  until  $t = \bar{t} \geq 1$ . In all  $t > \bar{t}$ , the economy will experience an equilibrium without process innovation effort. This, in turn, means that  $Y_t = 1 + \sigma t$  whenever  $t \leq \bar{t}$ , while  $Y_t = 1 + \bar{t}$  for all  $t > \bar{t}$ .*

Proposition 1 shows that, when  $\sigma \geq e^\eta - 1$  (i.e., when the increase in labor productivity generated by a process innovations is large enough relative to the disutility of R&D effort), the economy will keep investing in process innovation, as long as condition (9) keeps holding true. While  $t \leq \bar{t}$ , the economy will be on a positive growth path driven by rising labor productivity. This, in turn, translates into



households' incomes that are growing over time according to  $Y_t = 1 + \sigma t$ . However, once the economy reaches a stage where  $A_{t-1} > \bar{A}(\sigma)$ , the equilibrium will cease to exhibit thereafter any further positive process innovation effort. From then on, growth will stop forever, and  $Y_t$  will remain constant at  $Y_t = 1 + \bar{t}$ .

The intuition behind the fact that process innovation eventually comes to halt hinges on the decreasing marginal utility of consumption. When the household alive in period  $t$  is contemplating whether or not to sell the time endowment of  $R_t$  as R&D effort to a firm, the household faces a trade-off between higher consumption versus higher leisure. Setting  $\varepsilon_t = 1$  allows a level of consumption equal to  $Y_t(\varepsilon_t = 1) = 1 + \sigma A_{t-1} + \sigma$ . On the other hand,  $\varepsilon_t = 0$  leads a *lower* level of consumption,  $Y_t(\varepsilon_t = 0) = 1 + \sigma A_{t-1}$ , but it yields an *additional* non-pecuniary benefit,  $\eta$ . As the economy goes through subsequent rounds of process innovation, both  $Y_t(\varepsilon_t = 1)$  and  $Y_t(\varepsilon_t = 0)$  increase. Yet, in a context with decreasing marginal utility of consumption, higher values of  $Y_t(\varepsilon_t = 1)$  and  $Y_t(\varepsilon_t = 0)$  imply that the consumption gap between them becomes gradually less appealing relative to the required leisure sacrifice  $\eta$ .

#### 4 Introducing product innovations

We proceed now to introduce product innovation effort into the model. We consider the situation where the only quality version that was available before period  $t$  was the baseline quality. In this context, product innovation effort by  $R_t$  will allow introducing the quality version  $q_t = 1 + \rho$  to the market.

Before moving on to the full equilibrium analysis of the model, it proves convenient to first address the following two questions: *i*) What is the surplus that the product innovation yields to the firm that introduces it to the market? and *ii*) what is the price that the firm producing the quality version  $q_t = 1 + \rho$  will be able to charge for this commodity?

**Lemma 2** *Consider the researcher  $R_t$  alive in period  $t$  who is hired by a firm active in  $t$  to undertake product innovation effort. The surplus generated by the product innovation designed by  $R_t$  is given by:*

$$\pi_t^q = P_t \frac{1 + \sigma A_{t-1}}{1 + \rho} - (1 + \sigma A_{t-1}), \tag{11}$$

where  $P_t$  in (11) denotes the market price of the quality version  $q_t = 1 + \rho$ , which in equilibrium will be given by:

$$P_t = (1 + \rho) Y_t^{\rho/(1+\rho)}. \tag{12}$$

The first result in Lemma 2 shows the surplus (net of the labor cost) generated by a blueprint that may allow producing the higher-quality version ( $q_t = 1 + \rho$ ), given its price  $P_t$ . Notice that competition by firms for the researcher's effort time entails that, in an equilibrium with positive product innovation effort, the remuneration offered to  $R_t$  will have to be equal to the RHS of (11). The second result, displayed

by (12), shows that the price charged for the version of the good of quality  $1 + \rho$  rises with  $Y_t$ . The responsiveness of  $P_t$  to  $Y_t$  is a direct implication of our preference structure in (5), where the quality index  $q$  magnifies the utility derived from the physical quantity of consumption,  $x(q)$ . Such preference structure leads to a non-homothetic behavior where the willingness to pay for the version of the good with  $q = 1 + \rho$  rises with  $Y_t$ .<sup>16</sup>

The equilibrium value of  $P_t$  thus depends on the level of  $Y_t$ . In the model,  $Y_t$  is itself also an equilibrium object. In particular, the level of  $Y_t$  will be ultimately a function of the innovation effort equilibrium choices in  $t$ . Equation (8) states how  $Y_t$  rises with subsequent rounds of process innovation effort. The following lemma complements those results when we allow for product innovation effort as well.

**Lemma 3** *Consider an economy that has carried out  $A_{t-1}$  rounds of process innovations effort before period  $t$ , and which has never carried out any product innovation effort before  $t$ . The set of active firms  $\mathcal{F}_t$  will thus inherit a technology that will allow them to produce  $1 + \sigma A_{t-1}$  units of the baseline quality good with one unit of labor. If an active firm in  $t$  hires  $R_t$  to exert product innovation effort, in equilibrium, the total income in  $t$  will be given by:*

$$Y_t = (1 + \sigma A_{t-1})^{1+\rho}. \tag{13}$$

Lemma 3 shows that the larger the value of  $A_{t-1}$ , the greater  $Y_t$  will be in an equilibrium with product innovation effort. In addition, it is interesting to notice that when  $A_{t-1} = 0$ , expression (13) yields  $Y_t = 1$ . In other words, in the absence of *any* previous rounds of process innovations, product innovation effort cannot induce by itself a rise in incomes. As a result, the model requires that some initial spells of process innovation effort take place in order to ignite income growth, when starting off from the level generated by the prehistoric technology. In turn, as consumers' incomes grow, this may endogenously generate a change in relative profits between the types of innovation effort, and shift at some point R&D effort from process innovation to product innovation. We next study how this particular growth sequence may arise as an equilibrium outcome of the model.

Before moving on, some additional notation is needed to distinguish the type of innovation effort carried out. We use henceforth  $\varepsilon_{t,p} = 1$  for process innovation effort, and  $\varepsilon_{t,q} = 1$  for product innovation effort. We continue to denote by  $\varepsilon_t = 0$  the choice of consuming leisure.

<sup>16</sup> The equilibrium value of  $P_t$  in (12) is that one that leaves the household alive in  $t$  indifferent between buying the higher-quality version at the price  $P_t$  and buying the baseline quality version (which being the *numeraire* carries a price equal to one). Given our non-homothetic structure, this indifference price rises with  $Y_t$ .

### 4.1 Product innovation effort in a generic $t$

We first briefly show that an equilibrium with product innovation cannot arise in  $t = 1$ . After that, we proceed to study whether an equilibrium with product innovation effort may eventually arise along the growth path at some generic point in time  $t > 1$ .

In an equilibrium where  $\varepsilon_{1,q}^* = 1$  (i.e., with positive product innovation effort at time  $t = 1$ ) the income at  $t = 1$  would follow from (13) with  $A_0 = 0$ . This would yield  $Y_1 = 1$ . Using Lemma 2, we can observe that this would in turn imply that  $\pi_1^q = 0$ . As a consequence, no firm active in  $t = 1$  would be willing to pay anything in exchange for product innovation effort by the researcher alive in  $t = 1$ , and hence, no R&D effort would actually be offered to firms in the form of product innovation effort.

For an equilibrium with product innovation effort to arise in  $t$  two separate non-deviation conditions must be satisfied when  $\varepsilon_{t,q}^* = 1$  holds. The first is that the household must not prefer to deviate to consuming  $R_t$ 's time endowment as leisure. The second is that no firm active in  $t$  must find it profitable to request  $R_t$  to exert process innovation effort instead of product innovation effort. The previous paragraph shows that these non-deviation conditions fail to be satisfied at  $t = 1$ . The reason for this is that, in the absence of any previous process innovation effort (i.e., when  $A_{t-1} = 0$ ), equation (13) yields  $Y_t = 1$ , which (given our non-homothetic preference structure) turns out to be too small to turn product innovation effort profitable. However, according to (13), the level of  $Y_t$  in an equilibrium with product innovation effort is increasing in  $A_{t-1}$ . This suggests that there *may* exist a value of  $A_{t-1}$  large enough to be able to support such an equilibrium. The next lemma lays out this result formally.

**Lemma 4** *Consider an economy in a generic period  $t$ , such that  $q_{t-1} = 1$ , and which inherits a technology that allows firms in  $\mathcal{F}_t$  to transform one unit of labor time into  $1 + \sigma A_{t-1}$  units of the baseline quality variety. Suppose the researcher alive in  $t$  is offered a remuneration given by (11), with  $P_t$  given by (12), to exert product innovation effort for some firm  $f_{k,t} \in \mathcal{F}_t$ , and the worker alive in  $t$  is offered a wage given by (7). Then:*

(i) *If*

$$\rho \ln(1 + \sigma A_{t-1}) > \eta, \tag{14}$$

*the household alive  $t$  will prefer selling the R&D effort time to  $f_{k,t}$ , instead of deviating to consuming  $R_t$ 's time endowment as leisure.*

(ii) *If*

$$(1 + \sigma A_{t-1})^{1+\rho} > 1 + \sigma A_{t-1} + \sigma, \tag{15}$$

*no single firm in  $\mathcal{F}_t$  will find it profitable to request  $R_t$  to exert process innovation effort instead of product innovation effort.*

Lemma 4 presents the two non-deviation conditions that will be able to support an equilibrium with product innovation in  $t$ . These conditions will only hold when  $A_{t-1}$  is sufficiently large. In other words, unless the economy has previously undergone a sufficiently large number of rounds of process innovations, it will *not* be able to sustain an equilibrium with product innovation effort in  $t$ . The next proposition shows that, for a sufficiently large  $A_{t-1}$ , an economy where the only quality variety available before  $t$  was the baseline one will indeed exhibit an equilibrium with positive product innovation effort at time  $t$ .

**Proposition 2** *Consider an economy in period  $t$ , such that  $q_{t-1} = 1$ , and which has previously undergone  $A_{t-1}$  periods with positive process innovation effort. Then, the equilibrium in  $t$  will feature positive product innovation effort (i.e., in equilibrium,  $\varepsilon_{t,q}^* = 1$ ) if and only if both (14) and (15) hold true.*

Proposition 2 shows that when the two non-deviation conditions stipulated in Lemma 4 are satisfied, the economy will exhibit an equilibrium with product innovation effort at time  $t$ . In addition, this equilibrium is also unique. This implies that once an economy underwent enough rounds of process innovations in the past, so as the value of  $A_{t-1}$  that stems from (1) will be such that both (14) and (15) are satisfied, this economy will next switch at time  $t$  to an equilibrium where growth is pulled by product innovation effort. We next study whether such a switch will actually take place along the economy's growth path.

## 4.2 From process innovation to product innovation

Proposition 2 specifies the conditions that would lead to an equilibrium with positive product innovation effort in  $t$ . However, that proposition leaves one crucial question still unanswered: Whether or not the economy's growth path will actually lead eventually to a value of  $A_{t-1}$  large enough to make (14) and (15) hold simultaneously. In fact, if it fails to do so (because the incentives to keep undertaking further process innovation wane too quickly), the equilibrium with product innovation effort as described by Proposition 2 will actually *never* materialize. We proceed to study now the conditions required for a successful transition from a equilibrium with process innovation to one with product innovation at some point along the growth path.

Some additional notation will prove useful for future reference. Firstly, we will denote by  $\underline{A}_1$  the value of  $A_{t-1}$  that makes the LHS of (14) be equal to  $\eta$ . Namely:

$$\underline{A}_1 \equiv \frac{e^{\eta/\rho} - 1}{\sigma}. \tag{16}$$

Secondly, we will denote by  $\underline{A}_2$  the value of  $A_{t-1}$  that solves  $(1 + \sigma A_{t-1})^{1+\rho} = 1 + \sigma A_{t-1} + \sigma$ . In this case, there is no general explicit solution for  $\underline{A}_2$ , which is thus defined implicitly by:

$$\frac{(1 + \sigma \underline{A}_2)^{1+\rho}}{1 + \sigma \underline{A}_2 + \sigma} \equiv 1. \tag{17}$$

Both  $\underline{A}_1$  and  $\underline{A}_2$  are strictly decreasing in  $\sigma$ .<sup>17</sup> When  $A_{t-1} > \underline{A}_1$  and  $A_{t-1} > \underline{A}_2$ , the economy will feature an equilibrium with positive product innovation effort at time  $t$ . Since both conditions must hold for this, it is simpler to combine  $\underline{A}_1$  and  $\underline{A}_2$  together, and define:

$$\underline{A}(\sigma) \equiv \max\{\underline{A}_1, \underline{A}_2\}, \tag{18}$$

where in (18) we make the dependence of  $\underline{A}$  on  $\sigma$  explicit. The threshold function  $\underline{A}(\sigma)$  essentially pins down the value that  $A_{t-1}$  must necessarily reach for the economy to switch to an equilibrium with product innovation effort at time  $t$ .

By using the expressions in (16) and (17), we can pin down a cut-off level  $\hat{\sigma} > 0$  such that  $A(\sigma) \geq \underline{A}(\sigma)$  when  $\sigma \geq \hat{\sigma}$ , where recall that  $A(\sigma)$  is given by (10).

**Lemma 5** *Let*

$$\hat{\sigma} \equiv e^{n/\rho}(e^n - 1). \tag{19}$$

*When  $\sigma = \hat{\sigma}$ , the threshold functions (10) and (18) equal each other, that is,  $A(\hat{\sigma}) = \underline{A}(\hat{\sigma})$ . Furthermore,  $A(\sigma) > \underline{A}(\sigma)$  whenever  $\sigma > \hat{\sigma}$ , while  $A(\sigma) < \underline{A}(\sigma)$  whenever  $\sigma < \hat{\sigma}$ .*

Considering the result in Proposition 1, an interesting implication of the result in Lemma 5 follows quite straightforwardly: A sufficient condition for an economy to be able to reach a value of  $A_{t-1}$  above the threshold  $\underline{A}(\sigma)$  required for an equilibrium with product innovation effort to take place along its growth path is that  $\sigma$  is greater than  $\hat{\sigma}$ . Notice from (19) that  $\hat{\sigma}$  is strictly decreasing in  $\rho$  and strictly increasing in  $\eta$ . Intuitively, a greater  $\rho$  makes the marginal surplus generated by product innovation effort larger, which in turn reduces the minimum level of labor productivity beyond which an economy will be able to switch to an equilibrium with product innovation. On the other hand, a greater  $\eta$  countervails the effect of  $\rho$ , by increasing the implicit shadow price of innovation effort, thereby discouraging R&D effort.

The following proposition describes more formally the conditions under which a successful transition to a growth regime with product innovation effort will materialize along the economy's growth path.

**Proposition 3** *Let Assumption 3 hold true. Then, there exists a threshold level  $\tilde{\sigma}$ , where  $e^{n/\rho}(1 - e^{-n}) < \tilde{\sigma} < \hat{\sigma} \equiv e^{n/\rho}(e^n - 1)$ , such that:*

<sup>17</sup> The fact that  $\underline{A}_1$  decreases with  $\sigma$  can be observed directly from the expression in (16); a formal proof that  $\underline{A}_2$  decreases with  $\sigma$  is found in Online Appendix B.

- (i) If  $\sigma < \tilde{\sigma}$ , the economy will experience process innovation effort from  $t = 1$  until some period  $t = \bar{t} \geq 1$ , where  $\bar{t} \equiv \text{integer}\{\bar{A}(\sigma) + 1\}$  with  $\bar{A}(\sigma)$  given by (10), and it will stop carrying out any type of innovation effort for all  $t > \bar{t}$ .
- (ii) If  $\sigma \geq \tilde{\sigma}$ , the economy will experience process innovation effort from  $t = 1$  until some period  $t = \underline{t} \geq 1$ , where  $\underline{t} \equiv \text{integer}\{\underline{A}(\sigma) + 1\}$  with  $\underline{A}(\sigma)$  given by (18), and it will switch to an equilibrium with product innovation effort in  $t = \underline{t} + 1$

Proposition 3 shows that when  $\sigma$  is sufficiently low, the economy will never manage to switch to an equilibrium with product innovation. In these cases, process innovation and income growth will eventually come to a halt at  $t = \bar{t}$ . From then on, the value of  $A_{t-1}$  will remain constant at a level equal to  $A_{t-1} = \bar{t}$  for all  $t > \bar{t}$ . Incomes will, accordingly, also stay fixed thereafter at the level  $Y_t = 1 + \sigma\bar{t}$ .

On the other hand, for levels of  $\sigma$  that are large enough, an equilibrium with product innovation effort will actually arise at time  $t = \underline{t} + 1$ . In such cases, the economy will experience an initial phase of growth driven by process innovation effort from  $t = 1$  until  $t = \underline{t}$ . As incomes rise during this phase, the (implicit) willingness to pay for the higher-quality version increases. Eventually, at  $t = \underline{t}$ ,  $Y_t$  will have risen enough so as to increase the willingness to pay for  $q = 1 + \rho$  sufficiently to turn product innovation effort more profitable than process innovation effort in the following period. At this point, the growth-regime switch takes place at  $t = \underline{t} + 1$ .

It is interesting to highlight the impact of the different technological parameters in promoting a successful growth-regime switch along the growth path. Note from the result in Proposition 3 that a sufficient condition for this to take place is that  $\sigma \geq \hat{\sigma} \equiv e^{n/\rho}(e^n - 1)$ .<sup>18</sup> This condition essentially shows that both a greater  $\rho$  and a greater  $\sigma$  are instrumental for this growth-regime switch to materialize. The reason for the former is indeed quite straightforward: A greater  $\rho$  increases the marginal productivity of R&D effort in product innovation. The reason why a greater  $\sigma$  is also helpful for the growth-regime transition is, however, quite different and much more subtle. In this case, it is to do with the effect of non-homothetic preferences on the willingness to pay for quality upgrading. More precisely, since a greater  $\sigma$  implies that the income effect generated by each round of process innovation effort is larger, in a context with non-homotheticities along the quality dimension of consumption, this helps speeding up the transition to a regime with quality-upgrading innovations.

<sup>18</sup> Similarly, a sufficient condition for an economy not to be able to switch to an equilibrium with product innovation along the growth path is that  $\sigma \leq e^{n/\rho}(1 - e^{-n})$ . As a result, economies whose  $\sigma$  is too small, or whose  $\rho$  is too small, will tend to experience growth driven by process innovation during for some periods, after which growth will eventually come to a halt.

## 5 Quality upgrading and long-run growth

In our model, process innovation effort exhibits an inherent tendency to come to a halt. For this reason, as Sect. 4.2 shows, it becomes crucial that the switch to an equilibrium with product innovation effort takes place soon enough (or, otherwise, the growth-regime switch will simply end up not happening at all). Naturally, product innovation effort helps sustaining positive growth while it takes place. However, there is one additional positive effect that product innovation exerts on growth: It may also boost the incentives to further undertake process innovation effort in future periods.

Growth driven by process innovations is constantly at risk of being thwarted by its ensuing effect of the decreasing marginal utility of consumption. Product innovation effort works on a rather distinct dimension that does not face such a predicament: It spurs growth by allowing higher utility by each unit of physical consumption. Furthermore, the marginal utility of consumption declines more slowly for higher-quality versions than for lower quality ones. Quality upgrading thus relaxes the depressing effect that decreasing marginal utility of consumption imposes on the incentives to further raise physical production via process innovation. As a result, an economy that manages to switch at some point along its growth path to an equilibrium with product innovation effort, will also see the incentives to exert further R&D effort become reinvigorated by product quality upgrading. The following lemma shows an important preliminary result regarding the long-run growth path of an economy that manages to transition from a process to a product innovation regime.

**Lemma 6** *Consider an economy that satisfies  $\sigma \geq \tilde{\sigma}$ , where  $\tilde{\sigma}$  is the threshold defined in Proposition 3 beyond which the growth-regime switch will take place. This economy will be able to sustain an equilibrium with some type of innovation effort during the entire infinite time horizon  $\mathcal{T}$ .*

Lemma 6 essentially states that economies which are able to switch to an equilibrium with product innovation at time  $t = \underline{t} + 1$ , will also be able sustain an equilibrium with *some* type positive innovation effort at any time  $t' > \underline{t} + 1$ . We proceed to study now *which* type of innovation effort actually takes place along the economy's growth path.

**Proposition 4** *An economy will be able to sustain an equilibrium growth path with positive growth in the long run if and only if it satisfies  $\sigma \geq \tilde{\sigma}$ . Along such a growth path, the economy experiences the following growth sequence:*

1. *There is an initial growth phase driven by process innovation effort starting in  $t = 1$  until  $\underline{t} \geq 1$ , where  $\underline{t} \equiv \text{integer}\{\underline{A}(\sigma) + 1\}$  with  $\underline{A}(\sigma)$  given by (18).*
2. *At  $t = \underline{t} + 1$ , the economy switches to an equilibrium with product innovation effort. From  $t = \underline{t} + 1$  onward, the economy's growth path exhibits finite spells of*



*growth driven by process innovation effort, alternating with finite spells of growth driven by product innovation effort.*

Proposition 4 shows that economies that manage to sustain positive growth in the long run will exhibit finite spells of growth driven by process innovation effort, alternating with finite spells of growth driven product innovation. Those sequences of finite spells alternate each other indefinitely. This result showcases the interplay between process and product innovations present in our model. On the one side, the quantity expansion brought about by process innovations bolsters the incentives to start investing in quality-upgrading innovations. On the other side, the ensuing quality expansion stemming from product innovations relaxes the inherent tendency of profit prospects from further process innovations to decay. The alternation of equilibria with process and product innovation efforts exploits this feedback loop and is thus instrumental to keeping income growth alive in the long run.

An interesting implication of Proposition 4 is that the growth path of a successful economy will display deterministic growth cycles. In particular, spells where growth rates start to wane owing to the eroding effect of quantity expansion on marginal utility of consumption alternate with periods of higher growth rates in which quality upgrading takes place. In turn, higher growth rates resulting from quality upgrading will reinvigorate growth rates from further quantity expansion. This dynamic mechanism somehow resembles the one featured in Matsuyama (1999), which highlights the alternation of growth cycles driven by capital accumulation (to expand the stock of capital) and innovation activities (to expand the variety of capital goods). In Matsuyama (1999), the decreasing marginal productivity of a fixed variety of capital goods implies that expanding the variety of capital goods is necessary for sustaining long-run growth. In our model, cycles emerge from a different source of interaction: the complementarities of independently meaningful types of innovation effort, whose relative prominence shifts endogenously along the growth path.<sup>19</sup> Notice that an important difference between the two models is to do with the origin of the underlying tension leading to growth cycles in each of them. In Matsuyama (1999), cycles are the result of supply-side features of the economy. By contrast, the cycles described by Proposition 4 are crucially linked to the non-homothetic demand structure of the model. In particular, it is owing to such non-homothetic structure that quality upgrades from product innovations keep reviving the demand side of the economy, and thus regenerating the incentives to carry out process innovations.<sup>20</sup>

<sup>19</sup> Shleifer (1986) and Francois and Lloyd-Ellis (2003) also present models that feature deterministic growth cycles. In those models, cycles are the result of the endogenous clustering of innovations in certain moments of time due to the presence of (positive) demand externalities.

<sup>20</sup> Proposition 4 describes a growth path where the economy is moving back-and-forth between growth spells driven by process innovations and growth spells pulled by product innovations. However, the main intention of that result is not that of presenting those periodic discrete jumps as a truthful representation of the growth paths followed by developed economies, but to showcase the nature of the feedbacks between both types of innovation effort along the growth path as cleanly as possible.

## 5.1 Evolution of process and product innovation along the growth path

Proposition 4 shows that an economy featuring long-run growth undergoes an initial growth phase propelled by process innovation effort, followed by a second phase where growth is driven by product innovation effort. After this initial sequence, the economy alternates indefinitely between spells of growth pulled by process innovations and spells of growth pulled by product innovations. The proposition then leaves still unanswered one crucial question: Whether the preeminence of product innovations vis-a-vis process innovations rises along the growth path. The next proposition addresses this question. In particular, Proposition 5 shows that product innovations become increasingly prevalent along the growth, when considering a long enough time horizon.

**Proposition 5** *Consider an economy that satisfies the condition  $\sigma \geq \tilde{\sigma}$ , and which will therefore be able to sustain positive growth in the long run.*

*Let  $\mathcal{H}_T \mathfrak{S}\{1, 2, \dots, T\}$  denote the set of periods starting in  $t = 1$  and ending in  $t = T$ , and suppose that  $T$  is a large number.*

*Let  $\alpha_T \in (0, 1)$  denote the fraction of periods in  $\mathcal{H}_T$  that features an equilibrium with process innovation effort.*

*Let the time horizon of  $\mathcal{H}_T$  be extended by  $\Delta \geq 1$  additional periods, and denote the extended set of periods by  $\mathcal{H}_{T+\Delta} \mathfrak{S}\{1, 2, \dots, T + \Delta\}$ . Then, the fraction of periods in  $\mathcal{H}_{T+\Delta}$  that feature process innovation effort,  $\alpha_{T+\Delta} \in (0, 1)$ , will be smaller than in  $\mathcal{H}_T$  for  $\Delta$  large enough. In other words, there exists  $\tilde{\Delta} \geq 1$ , such that  $\alpha_{T+\Delta} < \alpha_T$  for any  $\Delta \geq \tilde{\Delta}$ .*

Proposition 5 states that equilibria exhibiting process innovation effort tend to occur less and less often along the growth path. This in turn means that periods where the equilibrium features product innovation effort will tend to be increasingly observed along the growth path. This last result then complements the dynamics described in Proposition 4 regarding the importance and interplay between process and product innovations. It shows that, while both types of innovations fulfill crucial roles in the model to sustain long-run growth, product innovations tend to become increasingly more prominent than process innovations as economies grow richer.

## 6 Concluding remarks

We presented a model where the combined impact of process and product innovations steer the economy along a growth path featuring both quantity and quality expansion. At early stages of development, when willingness to pay for quality upgrading is low, growth must be driven by the cost-cutting effect of process innovations. However, an economy cannot rely exclusively on process innovations

in order to achieve long-lasting growth, as their benefits tend to decrease as physical production keeps expanding, pushing individuals toward a state of relative satiation. Sustained growth necessitates thus that the economy becomes also able to generate product innovations as it moves along the development path, so as to overturn the tendency toward satiation. In addition, quality-upgrading innovations boost the incentives to keep expanding physical production. Therefore, while process innovations are necessary to turn product innovations sufficiently profitable, product innovations are able to regenerate profit prospects from further process innovations. This implicit feedback loop may keep growth alive in the long run.

Our model has restricted the consumption space to a very specific case: one single final good available in different quality versions, which are all perfect substitutes among each other. One important type of innovation effort that our model has then ruled out is that one that leads to a *horizontal* expansion in the set of final goods, as in Judd (1985), Romer (1990), and Grossman and Helpman (1991c, Ch.3), Young (1993). In principle, these types of innovations may also be able to keep growth alive in the long run. In particular, as profit prospects from cost-cutting innovations dwindle owing to decreasing marginal utility in a *given* good category, individuals may at some point find it worthwhile to introduce a completely *new* good category. This new final good would offer initially large profit prospects from process innovations, which would tend to diminish with subsequent rounds of it. We see this mechanism leading to a horizontal expansion of the set of consumption goods as complementary to the interplay between quantity and quality expansion studied by our model. Certainly, a model in which growth features a simultaneous expansion in quantity, quality, and variety of consumption, with positive feedbacks between all three dimensions, could yield a more encompassing description of growth in mature economies, and we see this as an appealing avenue of future research.

One other important limitation of the model is that firms and innovators are assumed to be alive for one period of time only. This simplification has two important implications. First, it cancels the effect that intertemporal externalities resulting from the interaction between process and product innovation could have on the decision to innovate in a given moment in time. Second, it removes any strategic consideration regarding creative destruction. Allowing for the positive effect of intertemporal externalities across types of innovation should help making the no-growth trap less likely to arise. On the other hand, the possibility of creative destruction may have the opposite effect. Incorporating these two dynamic considerations into the model could thus lead to smoother results in terms of growth dynamics and regime switches, possibly more aligned with those observed empirically.

Finally, our model has remained within a closed-economy framework. As a consequence, a crucial question it cannot address is whether international trade may somehow help spur growth in poorer economies by inducing quality-upgrading innovations there in order to cater to consumers located in richer economies. On the other hand, even within a context with open economies, the presence of trade costs, combined with non-homothetic preferences, may as well lead to a *home-market effect* similar to that one featured in Fajgelbaum et al. (2011) in a static framework, which could in turn push for dynamics in the opposite direction. In particular, a home-market effect stemming from non-homothetic demand schedules may lead to

dynamics where process innovation effort tends to gradually move to lower-income economies, while richer economies tend to increasingly specialize in generating product innovations. Studying the conditions under which either of these two conflicting forces seems to be the one that prevails would require a fully fledged endogenous growth model with open economies, which is a task we leave open for future research.

### Appendix A: Proofs

**Proof of Lemma 1** A household in  $t$  will optimally set  $\varepsilon_t^* = 1$  if and only if the utility obtained from consuming  $1 + \sigma A_{t-1} + \sigma$  units of  $q = 1$  is strictly greater than the utility derived from consuming  $1 + \sigma A_{t-1}$  units of it plus the utility of leisure,  $\eta$ . Using (8), together with the utility function (5) when  $Q_t = \{1\}$ , condition (9) obtains.  $\square$

**Proof of Proposition 1** Consider some generic period  $t = \tau - 1 \geq 1$ , such that  $A_{t-2} \leq \bar{A}(\sigma)$ . Notice then that the equilibrium in  $t = \tau - 1$  must feature  $\varepsilon_{\tau-1}^* = 1$ . Furthermore,  $\varepsilon_t^* = 1$  must have also held true for all  $t < \tau - 1$ . Based on this, consider then a continuous sequence of equilibria with positive process innovation before  $\tau$ , (1) implies that  $A_{\tau-1} = \tau - 1$ . Therefore, if  $\tau - 1 \leq \bar{A}(\sigma)$ , where  $\bar{A}(\sigma)$  is given by (10), it follows from Lemma 1 that the equilibrium in period  $\tau$  will feature  $\varepsilon_\tau^* = 1$ . On the other hand, if  $\tau - 1 > \bar{A}(\sigma)$ , Lemma 1 implies that the equilibrium in period  $\tau$  will feature  $\varepsilon_\tau^* = 0$ .  $\square$

**Proof of Lemma 2** We carry out the proof in three separate steps.

*Step 1)*  $P_t > (1 + \rho)Y_t^{\rho/(1+\rho)}$  cannot hold in equilibrium.

Using (5), we can observe that the utility obtained by the household alive in  $t$  if they chose to consume the version of the good with quality  $q = 1 + \rho$  would be given by:

$$U_t(q = 1 + \rho) = \ln \left[ (1 + \rho) \frac{Y_t}{P_t} \right]^{1+\rho}. \tag{20}$$

Instead, if they chose to consume the baseline quality version, they would obtain:

$$U_t(q = 1) = \ln (Y_t). \tag{21}$$

Comparing (20) and (21), we can observe that  $P_t > (1 + \rho)Y_t^{\rho/(1+\rho)}$  implies  $U_t(q = 1 + \rho) < U_t(q = 1)$ , and therefore, the household would not consume the higher-quality version.

*Step 2)*  $P_t < (1 + \rho)Y_t^{\rho/(1+\rho)}$  cannot hold in equilibrium.

Suppose in equilibrium the firm that hires  $R_t$  requests the researcher to exert product innovation effort, and that it sets  $P_t = \tilde{P}_t < (1 + \rho)Y_t^{\rho/(1+\rho)}$ . In equilibrium, firms

must necessarily satisfy the zero-profit condition. Hence, it must be the case that  $R_t$  is paid a remuneration equal to

$$\tilde{\pi}_t = \tilde{P}_t \frac{1 + \sigma A_{t-1}}{1 + \rho} - (1 + \sigma A_{t-1}).$$

Suppose now that some other active firm belonging to  $\mathcal{F}_t$  decided to offer  $R_t$  a remuneration  $\hat{\pi}_t$ , where

$$\hat{\pi}_t \equiv (\tilde{P}_t + \hat{\varepsilon}) \frac{1 + \sigma A_{t-1}}{1 + \rho} - (1 + \sigma A_{t-1}), \text{ and } \hat{\varepsilon} > 0.$$

This firm would then attract  $R_t$ . Furthermore, this firm could charge a price  $P'_t \equiv \tilde{P}_t + \varepsilon' < (1 + \rho)Y_t^{\rho/(1+\rho)}$ , where  $\varepsilon' > \hat{\varepsilon} > 0$ , for the higher-quality version of the final good, obtaining thus a positive profit. As a consequence, a situation where a firm charges a price  $P_t < (1 + \rho)Y_t^{\rho/(1+\rho)}$  for  $q = 1 + \rho$  while it also satisfies the zero profit condition cannot arise in equilibrium.

*Step 3)* Using again (20) and (21), we can first observe that when  $P_t = (1 + \rho)Y_t^{\rho/(1+\rho)}$  the household alive in  $t$  is indifferent between the baseline quality and the higher-quality version. Moreover, when (11) holds, there exists no profitable deviation to any firm  $f_{k,t} \in \mathcal{F}_t$ . In particular, in order to outcompete a firm whose strategy is characterized by (12) and (11), another firm should either offer the higher-quality version for a lower price or, alternatively, offer  $R_t$  a higher remuneration while keeping  $P_t = (1 + \rho)Y_t^{\rho/(1+\rho)}$  (since  $P_t > (1 + \rho)Y_t^{\rho/(1+\rho)}$  cannot hold in equilibrium, as shown by *Step 1*). Both deviations lead, however, to a loss.  $\square$

**Proof of Lemma 3** Using (11), we obtain that when  $R_t$  is hired to exert product innovation effort:

$$Y_t = P_t \frac{1 + \sigma A_{t-1}}{1 + \rho}. \tag{22}$$

Replacing (22) into (12) yields  $P_t = (1 + \rho)[P_t(1 + \sigma A_{t-1})(1 + \rho)^{-1}]^{\frac{\rho}{1+\rho}}$ , from where we may solve for  $P_t$  and obtain:

$$P_t = (1 + \rho)(1 + \sigma A_{t-1})^\rho. \tag{23}$$

Lastly, plugging (23) into (22) yields (13).  $\square$

**Proof of Lemma 4** Part *i*). First of all, notice that using (11) and (13), it follows that when all  $R_t$  exerts product innovation effort, the level of utility achieved by the household alive in  $t$  will be given by

$$\begin{aligned}
 &U_t(\varepsilon_{t,q} = 1 | \omega_t = 1 + \sigma A_{t-1}, r_t = (1 + \sigma A_{t-1})^{1+\rho} - \omega_t, P_t) \\
 &= \ln \left[ (1 + \rho) \frac{(1 + \sigma A_{t-1})^{1+\rho}}{P_t} \right]^{1+\rho},
 \end{aligned}
 \tag{24}$$

Plugging (12), together with  $Y_t = (1 + \sigma A_{t-1})^{1+\rho}$ , into (24) yields:

$$U_t(\varepsilon_{t,q} = 1) = (1 + \rho) \ln (1 + \sigma A_{t-1}).
 \tag{25}$$

Suppose now this household would deviate from  $\varepsilon_{t,q} = 1$ , within that context, to  $\varepsilon_t = 0$ . In this case,  $Y_t(\varepsilon_t = 0) = \omega_t = 1 + \sigma A_{t-1}$ . Recall that the price (12) leaves indifferent a household with  $Y_t = (1 + \sigma A_{t-1})^{1+\rho}$  between the two versions of the consumption good. Since,  $1 + \sigma A_{t-1} < (1 + \sigma A_{t-1})^{1+\rho}$ , it must then be the case that, if setting  $\varepsilon_t = 0$ , the household alive at time  $t$  will then strictly prefer consuming the baseline quality rather than (the more expensive) higher-quality version,  $q = 1 + \rho$ . Moreover, when  $\omega_t = 1 + \sigma A_{t-1}$ , there will always be some firm belonging to  $\mathcal{F}_t$  willing to offer the baseline quality version, as it would break even by doing so. Hence, when the household alive in  $t$  sets  $\varepsilon_t = 0$ , within the described context, it will achieve as utility:

$$\begin{aligned}
 &U_t(\varepsilon_t = 0 | \omega_t = 1 + \sigma A_{t-1}, r_t = (1 + \sigma A_{t-1})^{1+\rho} - \omega_t, P_t = (1 + \rho)(1 + \sigma A_{t-1})^\rho) \\
 &= \ln (1 + \sigma A_{t-1}) + \eta.
 \end{aligned}
 \tag{26}$$

Finally, comparing (25) and (26), condition (14) ensures that  $U_t(\varepsilon_{t,q} = 1 | \cdot) > U_t(\varepsilon_t = 0 | \cdot)$ , completing the proof.

Part ii) Notice first that a firm belonging to  $\mathcal{F}_t$  must offer at least a remuneration equal to  $r_t = (1 + \sigma A_{t-1})^{1+\rho} - \omega_t$ , with  $\omega_t = 1 + \sigma A_{t-1}$ , if it wishes to hire  $R_t$  and request R&D effort directed to process innovation from him. Hence, the surplus a firm belonging to  $\mathcal{F}_t$  would obtain from requesting process innovation effort from  $R_t$  would be equal to  $\sigma - r_t$ , which using  $r_t = (1 + \sigma A_{t-1})^{1+\rho} - \omega_t$  and  $\omega_t = 1 + \sigma A_{t-1}$ , yields  $\sigma - r_t = (1 + \sigma A_{t-1} + \sigma) - (1 + \sigma A_{t-1})^{1+\rho}$ . Given that (15) implies  $(1 + \sigma A_{t-1} + \sigma) - (1 + \sigma A_{t-1})^{1+\rho} < 0$ , no firm active in  $t$  would, in this context, prefer to deviate to requesting process innovation effort from  $R_t$ .  $\square$

**Proof of Proposition 2** Notice first that in an equilibrium with  $\varepsilon_{t,q}^* = 1$ , the remuneration received by  $R_t$  will be equal to  $(1 + \sigma A_{t-1})^{1+\rho} - (1 + \sigma A_{t-1})$ . Notice also that, when  $q_{t-1} = 1$ , the surplus generated by a process innovation is equal to  $\sigma$ . As a result, if  $\sigma > (1 + \sigma A_{t-1})^{1+\rho} - (1 + \sigma A_{t-1})$ , an equilibrium with  $\varepsilon_{t,q}^* = 1$  cannot arise. On the other hand, if  $\sigma < (1 + \sigma A_{t-1})^{1+\rho} - (1 + \sigma A_{t-1})$ , the non-deviation condition (15) will be verified in an equilibrium with  $\varepsilon_{t,q}^* = 1$ . Furthermore,  $\sigma < (1 + \sigma A_{t-1})^{1+\rho} - (1 + \sigma A_{t-1})$  implies that an equilibrium with  $\varepsilon_{t,p}^* = 1$  cannot

arise either, as the deviation to  $\varepsilon_{t,q}^* = 1$  becomes profitable to firms. Secondly, notice that using (12) and (13), from (5) it follows that the utility of the household alive in  $t$  when  $\varepsilon_{t,q}^* = 1$  will be given by  $U_t(\varepsilon_{t,q}^* = 1) = (1 + \rho) \ln(1 + \sigma A_{t-1}) - \eta$ , while the utility it would obtain if it deviated to  $\varepsilon_t = 0$  would be  $U_t(\varepsilon_t = 0) = \ln(1 + \sigma A_{t-1})$ . Comparing these two expressions, we can observe that (14) implies that such a deviation would not be profitable, while if it that condition failed to hold the household would rather set  $\varepsilon_t = 0$ . Finally, from a situation in which  $\varepsilon_t = 0$ , a firm could offer a remuneration to  $R_t$  high enough to induce the household to sell their R&D time endowment to the firm, which would in turn use it for product innovation effort so long as (14) holds true. Hence, there cannot exist an equilibrium with  $\varepsilon_t^* = 0$  when (14) holds true.  $\square$

**Proof of Lemma 5** Equalizing (10) and (16) yields to  $\bar{A}(\sigma) = \underline{A}_1(\sigma)$  if and only if  $\sigma = \hat{\sigma} \equiv e^{n/\rho}(e^n - 1)$ . Next, plugging this value into either (10) or (16) yields:

$$\bar{A}(\hat{\sigma}) = \underline{A}_1(\hat{\sigma}) = \frac{e^{n/\rho} - 1}{e^{n/\rho}(e^n - 1)}. \tag{27}$$

Finally, plugging  $\sigma = \hat{\sigma}$  and  $\underline{A}_2 = (e^{n/\rho} - 1) / [e^{n/\rho}(e^n - 1)]$  into the LHS of (17), one can verify that equation (17) holds true for those values of  $\sigma$  and  $\underline{A}_2$ . As a result, it follows that  $\bar{A}(\hat{\sigma}) = \underline{A}_1(\hat{\sigma}) = \underline{A}_2(\hat{\sigma}) = \underline{A}(\hat{\sigma})$ , where  $\hat{\sigma} = e^{n/\rho}(e^n - 1)$ . In addition, given that  $\bar{A}'(\sigma) > 0$  and  $\underline{A}'(\sigma) < 0$ , it also follows that  $\bar{A}(\sigma) \geq \underline{A}(\sigma)$  whenever  $\sigma \geq \hat{\sigma}$ .  $\square$

**Proof of Proposition 3** Notice first that, whenever  $\sigma < \hat{\sigma}$ , using (16) and (17) we have that  $\underline{A}(\sigma) = \underline{A}_1$ . Hence, using (10) and (16), we can observe that whenever  $\sigma < \hat{\sigma}$ , we have that  $\underline{A}(\sigma) - \bar{A}(\sigma) = (e^{n/\rho}/\sigma) - (e^n - 1)^{-1}$ . Since  $\bar{t} \equiv \text{integer}\{1 + \bar{A}(\sigma)\} \leq 1 + \bar{A}(\sigma)$ , then the economy will not be able to switch to an equilibrium with product innovation when  $(e^{n/\rho}/\sigma) - (e^n - 1)^{-1} \geq 1$ , from where it follows that a sufficient condition for this switch not to take place is that  $\sigma \leq e^{n/\rho}(1 - 1)e^{-n}$ . Since  $\sigma \geq \hat{\sigma}$  is a sufficient condition for  $A_{\bar{t}} > \underline{A}(\sigma)$ , a regime switch would take place no later than in  $\bar{t} + 1$  when  $\sigma \geq \hat{\sigma}$  holds true. As a consequence, by continuity, there must exist some  $\tilde{\sigma}$ , such that  $e^{n/\rho}(1 - 1)e^{-n} < \tilde{\sigma} < \hat{\sigma}$ , below which the economy will never reach a situation where  $A_{\bar{t}-1} \geq \underline{A}(\sigma)$ , and the regime switch will never take place along the growth path. By using the same reasoning, it follows as well that when  $\sigma$  lies above  $\tilde{\sigma}$ , we have that  $\bar{t} \equiv \text{integer}\{1 + \bar{A}(\sigma)\} > \underline{A}(\sigma)$ , and the economy will thus be able to switch at some point to an equilibrium with product innovation effort in a period no later than  $t = \bar{t} + 1$ , given by  $\underline{t} + 1$ , where  $\underline{t} \equiv \text{integer}\{1 + \underline{A}(\sigma)\}$ .  $\square$

**Proof of Lemma 6** First of all, notice that when  $\sigma \geq \tilde{\sigma}$ , Proposition 3 implies that, in equilibrium, the economy will experience process innovation effort during  $t \leq \underline{t}$ , and product innovation effort at  $t = \underline{t} + 1$ . The rest of the proof will show that for any  $t' > \underline{t} + 1$  the action  $\varepsilon_{t',q} = 1$  will dominate  $\varepsilon_{t'} = 0$ .



*Step 1. Generalization of Lemma 2 for any  $q \in Q$ :* The expected return of the product innovation blueprint designed by a generic  $R_t$  alive in  $t$  will be:

$$\pi_t^q = P_t(q_t) \frac{1 + \sigma A_t}{q_t} - \omega_t, \tag{28}$$

where  $P_t(q_t)$  is the price of the (newly designed) quality version  $q_t \in Q$ . To compute the equilibrium value of  $P_t(q_t)$ , notice that, based on (5), this will follow from the condition

$$\ln \left( q_t \frac{Y_t}{P_t(q_t)} \right)^{q_t} \geq \ln(Y_t), \tag{29}$$

from where we can obtain

$$P_t(q_t) = q_t Y_t^{(q_t-1)/q_t} \tag{30}$$

when (29) holds with equality. Lastly, the fact that all firms active in  $t$  inherit a technology that allows producing  $(1 + \sigma A_{t-1})/q_{t-1}$  units of the quality version  $q_{t-1} \in Q$ , where  $q_{t-1} = q_t - \rho$ , with one unit of labor in turn implies that:

$$\omega_t = P_t(q_t) \frac{1 + \sigma A_{t-1}}{q_{t-1}} = Y_t^{(q_{t-1}-1)/q_{t-1}} (1 + \sigma A_{t-1}). \tag{31}$$

*Step 2. Generalization of Lemma 3 for any  $q \in Q$ :* Consider an equilibrium in a generic time  $t$  where  $\varepsilon_{t,q}^* = 1$ . Then, using (28), (30), and (31), we obtain:

$$Y_t(\varepsilon_{t,q} = 1) = (1 + \sigma A_{t-1})^{q_t}. \tag{32}$$

*Step 3.* Consider now some moment in time  $t' > \underline{t} + 1$ . Due to Proposition 3, when  $\sigma \geq \tilde{\sigma}$ , we must have that  $A_{t'-1} \geq \underline{t} \equiv \text{integer}\{A(\sigma) + 1\}$  and  $q_{t'-1} \geq 1 + \rho$ . Suppose also that  $\varepsilon_{t,q} = 1$  in each period  $t \geq \underline{t} + 1$ . Then, using (32):

$$Y_{t'}(\varepsilon_{t',q} = 1) = (1 + \sigma \underline{t})^{q_{t'}}. \tag{33}$$

Using next the utility function (5), together with (33) and (30), we may obtain:

$$U_{t'}(\varepsilon_{t',q} = 1) = q_{t'} \ln (1 + \sigma \underline{t}), \tag{34}$$

which denotes the level of utility achieved by a generic household alive in  $t = t'$  when  $\varepsilon_{t,q} = 1$  for all  $t \geq \underline{t} + 1$ . On the other hand, if in such same circumstance  $R_{t'}$  deviates to  $\varepsilon_{t'} = 0$ , the household alive in  $t'$  will achieve a utility level given by:

$$U_{t'}(\varepsilon_{t'} = 0) = (q_{t'} - \rho) \ln (1 + \sigma \underline{t}) + \eta. \tag{35}$$

Comparing (34) and (35), we obtain:

$$U_{t'}(\varepsilon_{t',q} = 1) > U_{t'}(\varepsilon_{t'} = 0) \iff \rho \ln (1 + \sigma \underline{t}) > \eta. \tag{36}$$

But now notice that  $\rho \ln(1 + \sigma \underline{\sigma}) > \eta$  actually holds true for any economy that verifies  $\sigma > \tilde{\sigma}$ , completing the proof.  $\square$

**Proof of Proposition 4 Part 1.** The fact that there is an initial growth phase between  $t = 1$  and  $t = \underline{t} \geq 1$  follows from Proposition 3.

**Part 2.** The fact that there is a second growth phase starting in  $t = \underline{t} + 1$  that is driven by product innovation effort also follows from Proposition 3. Next, to prove that this growth phase lasts for a finite number of periods, we proceed by contradiction. Suppose that the only type of innovation effort undertaken during  $t > \underline{t} + 1$  is in product innovation. In this case, we would have  $A_{t-1} = \underline{t}$  for all  $t > \underline{t} + 1$ . In addition, the highest quality version available in periods  $t > \underline{t} + 1$  would be  $q_t = 1 + (t - \underline{t})\rho$ . As a consequence, when  $\varepsilon_{t,q} = 1$  for all  $t > \underline{t} + 1$ , the expected return generated by a product innovation blueprint (net of wages,  $\omega_t$ ) will be

$$\pi_t^q(\tilde{\varepsilon}_{q,t>\underline{t}+1}) = (1 + \sigma \underline{\sigma})^{1+(t-\underline{t})\rho} - \omega_t, \tag{37}$$

where we use  $\tilde{\varepsilon}_{q,t>\underline{t}+1}$  to denote the *hypothetical* path in which  $\varepsilon_{t,q} = 1$  for all  $t > \underline{t} + 1$ . For this to be an equilibrium,  $\pi_t^q(\tilde{\varepsilon}_{q,t>\underline{t}+1})$  should be larger than the expected return obtained when shifting  $R_t$  alive in a generic  $t > \underline{t} + 1$  to setting  $\varepsilon_{t,p} = 1$ , which would yield:

$$\pi_t^p(\varepsilon_{t,p} = 1) = Y_t^{(t-1-t)\rho/[1+(t-1-t)\rho]} (1 + \sigma \underline{\sigma} + \sigma) - \omega_t. \tag{38}$$

Comparing (37) and (38), we can observe that  $\pi_t^q(\tilde{\varepsilon}_{q,t>\underline{t}+1}) > \pi_t^p(\varepsilon_{t,p} = 1)$  requires  $(1 + \sigma \underline{\sigma})^{1+(t-\underline{t})\rho} \geq (1 + \sigma \underline{\sigma} + \sigma)^{1+[(t-1)-\underline{t}]\rho}$ , which will fail to hold true when  $t$  becomes sufficiently large (i.e., when  $t$  departs sufficiently from  $\underline{t}$ ).

**Part 3.** Lastly, to prove that after  $t = \hat{t} \geq \underline{t} + 1$  the economy will be able to sustain positive growth forever by alternating finite spells where the equilibrium features process innovation effort with finite spells where the equilibrium features product innovation effort, we proceed by again contradiction, while bearing in mind the result in Lemma 6. Consider first the case of a hypothetical economy that for all periods  $t \geq t'$  features process innovation effort in equilibrium, where we let  $t' > \hat{t}$ . In that case, we will have  $A_t = t - (q_{t'-1} - 1)/\rho$  and  $q_t = q_{t'-1}$ , for all  $t \geq t'$ . The expected returns of a process innovation blueprint in given period  $t > t'$  will be

$$\pi_t^p(\tilde{\varepsilon}_{p,t \geq t'}) = \{1 + \sigma [t - (q_{t'-1} - 1)/\rho]\}^{q_{t'-1}} - \omega_t, \tag{39}$$

where  $\tilde{\varepsilon}_{p,t \geq t'}$  denotes the *hypothetical* growth path in which  $\varepsilon_{t,p} = 1$  for all  $t \geq t'$ . For this to be an equilibrium,  $\pi_t^p(\tilde{\varepsilon}_{p,t \geq t'})$  should be larger than the expected return obtained when shifting researcher  $R_t$  alive in a generic  $t \geq t'$  to setting  $\varepsilon_{t,q} = 1$ , which would yield

$$\pi_t^q(\varepsilon_{t,q} = 1) = Y_t^{(q_{t'-1} + \rho - 1)/(q_{t'-1} + \rho)} \{1 + \sigma [t - 1 - (q_{t'-1} - 1)/\rho]\} - \omega_t. \tag{40}$$

Comparing (39) and (40), it follows that  $\pi_t^p(\tilde{\varepsilon}_{p,t \geq t'}) \geq \pi_t^q(\varepsilon_{t,q} = 1)$  requires

$$\{1 + \sigma [t - (q_{t'-1} - 1)/\rho]\}^{q_{t'-1}} > \{1 + \sigma [t - 1 - (q_{t'-1} - 1)/\rho]\}^{q_{t'-1} + \rho}$$

which will fail to hold when  $t$  becomes sufficiently large. As a consequence, the economy cannot possibly sustain an equilibrium growth path with only process innovation effort over an infinitely long sequence of consecutive periods.

Consider next the case of a hypothetical economy that for all periods  $t \geq t'$  features product innovation effort in equilibrium, where again we let  $t' > \hat{t}$ . In this case,  $A_{t-1} = (t' - 1) - (q_{t'-1} - 1)/\rho$  and  $q_t = q_{t'-1} + (t - t' + 1)\rho$ , for all  $t \geq t'$ . These results in turn imply that in a generic period  $t \geq t'$  we will have:

$$\pi_t^q(\tilde{\epsilon}_{q,t \geq t'}) = \{1 + \sigma [(t' - 1) - (q_{t'-1} - 1)/\rho]\}^{q_{t'-1} + (t - t' + 1)\rho} - \omega_t, \tag{41}$$

where we use  $\tilde{\epsilon}_{q,t \geq t'}$  to denote the *hypothetical* path in which  $\epsilon_{t,q} = 1$  for all  $t \geq t'$ . For this to be an equilibrium,  $\pi_t^q(\tilde{\epsilon}_{q,t \geq t'})$  should be larger than the expected return obtained when shifting researcher  $R_t$  alive in a generic  $t \geq t'$  to setting  $\epsilon_{t,p} = 1$ , which would yield:

$$\pi_t^p(\epsilon_{t,p} = 1) = Y_t^{[q_{t'-1} + (t - t')\rho - 1]/[q_{t'-1} + (t - t')\rho]} \{1 + \sigma [t' - (q_{t'-1} - 1)/\rho]\} - \omega_t. \tag{42}$$

Comparing (41) and (42), it follows that  $\pi_t^q(\tilde{\epsilon}_{q,t \geq t'}) > \pi_t^p(\epsilon_{t,p} = 1)$  requires

$$\{1 + \sigma [(t' - 1) - (q_{t'-1} - 1)/\rho]\}^{q_{t'-1} + (t - t' + 1)\rho} \geq \{1 + \sigma [t' - (q_{t'-1} - 1)/\rho]\}^{q_{t'-1} + (t - t')\rho}$$

which will fail to hold when  $t$  becomes sufficiently large. As a result, the economy cannot possibly sustain an equilibrium growth path with only product innovation effort over an infinitely long sequence of consecutive periods.

The previous two contradictions imply thus that there cannot exist an equilibrium growth path featuring either infinitely long spells of process innovation effort or infinitely long spells of product innovation effort. Lemma 6 stipulates that an economy satisfying  $\sigma \geq \tilde{\sigma}$  will always be able to sustain an equilibrium with some type of innovation effort. Hence, it must be the case that the growth path will necessarily exhibit finite spells with process innovation effort in equilibrium, alternating with finite spells with product innovation effort in equilibrium.  $\square$

**Proof of Proposition 5** When  $T$  is a large number and the fraction of periods in  $\mathcal{H}_T$  featuring an equilibrium with  $\epsilon_{t,p}^* = 1$  is given by  $\alpha_T$ , we have that:

$$q_T = 1 + (1 - \alpha_T)T\rho, \tag{43}$$

$$A_T = \alpha_T T. \tag{44}$$

Using (43) and (44), we can observe that the equilibrium in  $T + 1$  will feature  $\epsilon_{T+1,p}^* = 1$  if  $(1 + \alpha_T T\sigma)^{1 + (1 - \alpha_T)T\rho + \rho} < (1 + \alpha_T T\sigma + \sigma)^{1 + (1 - \alpha_T)T\rho}$  holds true, while the economy will have an equilibrium with  $\epsilon_{T+1,q}^* = 1$  when  $(1 + \alpha_T T\sigma)^{1 + (1 - \alpha_T)T\rho + \rho} > (1 + \alpha_T T\sigma + \sigma)^{1 + (1 - \alpha_T)T\rho}$ . The previous two inequalities can be combined, to obtain

$$\begin{aligned} \varepsilon_{T+1,q}^* &= 1 \quad \text{if } \Gamma(\alpha_T, T) > Y(\alpha_T, T), \\ \varepsilon_{T+1,p}^* &= 1 \quad \text{if } \Gamma(\alpha_T, T) < Y(\alpha_T, T), \end{aligned} \quad (45)$$

where

$$\Gamma(\alpha_T, T) \equiv 1 + \frac{\rho}{1 + (1 - \alpha_T)T\rho} \quad \text{and} \quad Y(\alpha_T, T) \equiv \frac{\ln(1 + \alpha_T T\sigma + \sigma)}{\ln(1 + \alpha_T T\sigma)}. \quad (46)$$

Notice, first, from (46) that  $\partial\Gamma(\cdot)/\partial\alpha_T > 0$ ,  $\partial Y(\cdot)/\partial\alpha_T < 0$ ,  $\partial\Gamma(\cdot)/\partial T < 0$ , and  $\partial Y(\cdot)/\partial T < 0$ . It can also be shown (see proof in Online Appendix B) that, there exists a finite value  $\hat{T}(\alpha_T)$ , such that  $\Gamma(\alpha_T, \hat{T}(\alpha_T)) = Y(\alpha_T, \hat{T}(\alpha_T))$ , and  $\Gamma(\alpha_T, T) < Y(\alpha_T, T)$  for  $T < \hat{T}$  while  $\Gamma(\alpha_T, T) > Y(\alpha_T, T)$  for  $T > \hat{T}$ .

Suppose now that after extending the horizon from  $T$  to  $T + \Delta$ , the fraction of periods in which the equilibrium exhibits  $\varepsilon_{t,p}^* = 1$  remains equal to  $\alpha_T$ . Then, there will be some  $\tilde{\Delta}$  such that for all  $\Delta > \tilde{\Delta}$ , we will have that  $\Gamma(\alpha_T, T + \Delta) > Y(\alpha_T, T + \Delta)$ . As a consequence, the fraction of periods in which  $\varepsilon_{t,p}^* = 1$  cannot remain indefinitely equal to  $\alpha_T$ . Furthermore, since  $\partial\Gamma(\cdot)/\partial\alpha_T > 0$  and  $\partial Y(\cdot)/\partial\alpha_T < 0$ , it must then be the case that  $\alpha_{T+\Delta} < \alpha_T$  for  $\Delta > \tilde{\Delta}$ , where  $\tilde{\Delta}$  is a finite positive value.  $\square$

## References

- Aghion, P., Howitt, P.: A model of growth through creative destruction. *Econometrica* **60**, 323–351 (1992)
- Aghion, P., Howitt, P.: Research and development in the growth process. *J. Econ. Growth* **1**, 49–73 (1996)
- Akcigit, U., Kerr, W.: Growth through heterogeneous innovations. *J. Polit. Econ.* **126**, 1374–1443 (2018)
- Argente, D., Lee, M., Moreira, S.: Innovation and product reallocation in the great recession. *J. Monet. Econ.* **93**, 1–20 (2018)
- Argente, D., Lee, M., Moreira, S.: How Do Firms Grow? The Life Cycle of Products Matters. Working Paper (2019)
- Bas, M., Strauss-Kahn, V.: Input-trade liberalization, export prices and quality upgrading. *J. Int. Econ.* **95**, 250–262 (2015)
- Berli, A., Weiss, F., Zilibotti, F., Zweimüller, J.: Demand forces of technical change: evidence from the Chinese manufacturing industry. *China Econ. Rev.* **60**, 101157 (2020)
- Bils, M., Klenow, P.: Quantifying quality growth. *Am. Econ. Rev.* **91**, 1006–1030 (2001)
- Brambilla, I., Lederman, D., Porto, G.: Exports, export destinations, and skills. *Am. Econ. Rev.* **102**, 3406–3438 (2012)
- Bresnahan, T., Trajtenberg, M.: General purpose technologies ‘Engines of growth’? *J. Econom.* **65**, 83–108 (1995)
- Choi, Y.C., Hummels, D., Xiang, C.: Explaining import quality: the role of the income distribution. *J. Int. Econ.* **77**, 265–275 (2009)
- Chu, A.C., Cozzi, G., Galli, S.: Does intellectual monopoly stimulate or stifle innovation? *Eur. Econ. Rev.* **56**, 727–746 (2012)
- Cohen, W.M., Klepper, S.: Firm size and the nature of innovation within industries: the case of process and product R&D. *Rev. Econ. Stat.* **78**, 232–243 (1996)
- Fajgelbaum, P., Grossman, G., Helpman, E.: Income distribution, product quality, and international trade. *J. Polit. Econ.* **119**, 721–765 (2011)
- Fajgelbaum, P., Grossman, G., Helpman, E.: A Linder hypothesis for foreign direct investment. *Rev. Econ. Stud.* **82**, 83–121 (2015)
- Flach, L.: Quality upgrading and price heterogeneity: evidence from Brazilian exporters. *J. Int. Econ.* **102**, 282–290 (2016)

- Flam, H., Helpman, E.: Vertical product differentiation and north–south trade. *Am. Econ. Rev.* **77**, 810–822 (1987)
- Foellmi, R., Zweimüller, J.: Income distribution and demand-induced innovations. *Rev. Econ. Stud.* **73**, 941–960 (2006)
- Foellmi, R., Wüergler, T., Zweimüller, J.: The macroeconomics of model T. *J. Econ. Theory* **153**, 617–647 (2014)
- Francois, P., Lloyd-Ellis, H.: Animal spirits through creative destruction. *Am. Econ. Rev.* **93**, 530–550 (2003)
- Garcia-Macia, D., Hsieh, C., Klenow, P.: How destructive is innovation. *Econometrica* **87**, 1507–1541 (2019)
- Grossman, G., Helpman, E.: Quality ladders in the theory of growth. *Rev. Econ. Stud.* **5**(8), 43–61 (1991a)
- Grossman, G., Helpman, E.: Quality ladders and product cycles. *Q. J. Econ.* **106**, 557–586 (1991b)
- Grossman, G., Helpman, E.: *Innovation and Growth in the Global Economy*. MIT Press, Cambridge (1991c)
- Hallak, J.C.: Product quality and the direction of trade. *J. Int. Econ.* **68**, 238–265 (2006)
- Harrison, R., Jaumandreu, J., Mairesse, J., Peters, B.: Does innovation stimulate employment? A firm-level analysis using comparable micro-data from four European countries. *Int. J. Ind. Org.* **35**, 29–43 (2014)
- Helpman, E., Trajtenberg, M.: Diffusion of general purpose technologies. In: Helpman, E. (ed.) *General Purpose Technologies and Economic Growth*. MIT Press, Cambridge (1998)
- Hottman, C., Redding, S., Weinstein, D.: Quantifying the sources of firm heterogeneity. *Q. J. Econ.* **131**, 1291–1364 (2016)
- Huergo, E., Jaumandreu, J.: How does probability of process innovation change with firm age? *Small Bus. Econ.* **22**, 193–207 (2004)
- Jaimovich, E., Merella, V.: Quality ladders in a Ricardian model of trade with nonhomothetic preferences. *J. Eur. Econ. Assoc.* **10**, 908–937 (2012)
- Jaimovich, E., Merella, V.: Love for quality, comparative advantage, and trade. *J. Int. Econ.* **97**, 376–391 (2015)
- Jones, C.I.: R&D-based models of economic growth. *J. Polit. Econ.* **103**, 759–784 (1995)
- Jovanovic, B., Nyarko, Y.: Learning by doing and the choice of technology. *Econometrica* **64**, 1299–1310 (1996)
- Jovanovic, B., Rob, R.: Long waves and short waves: growth through intensive and extensive search. *Econometrica* **58**, 1391–1409 (1990)
- Judd, K.L.: On the performance of patents. *Econometrica* **53**, 567–585 (1985)
- Kolb, R., Hoover, M.: *The History of Quality in Industry*. Sandia Report (2012)
- Kortum, S.: Research, patenting, and technological change. *Econometrica* **65**, 1389–1419 (1997)
- Manova, K., Zhang, Z.: Export prices across firms and destinations. *Q. J. Econ.* **127**, 379–436 (2012)
- Marsiglio, S., Tolotti, M.: Endogenous growth and technological progress with innovation driven by social interactions. *Econ. Theory* **65**, 293–328 (2018)
- Matsuyama, K.: Growing through cycles. *Econometrica* **67**, 335–347 (1999)
- Matsuyama, K.: The rise of mass consumption societies. *J. Polit. Econ.* **110**, 1035–1070 (2002)
- Murphy, K., Shleifer, A.: Quality and trade. *J. Dev. Econ.* **53**, 1–15 (1997)
- Parisi, M., Schiantarelli, F., Sembenelli, A.: Productivity, innovation and R&D: micro evidence for Italy. *Eur. Econ. Rev.* **50**, 2037–2061 (2006)
- Peters, B., Roberts, M., Vuong, V., Fryges, H.: Estimating dynamic R&D choice: an analysis of costs and long-run benefits. *RAND J. Econ.* **48**, 409–437 (2017)
- Peretto, P.: Technological change, market rivalry, and the evolution of the capitalist engine of growth. *J. Econ. Growth* **3**, 53–80 (2007)
- Peretto, P., Connolly, M.: The manhattan metaphor. *J. Econ. Growth* **12**, 329–350 (2007)
- Redding, S.: Path dependence, endogenous innovation, and growth. *Int. Econ. Rev.* **43**, 1215–1248 (2002)
- Romer, P.: Endogenous technological change. *J. Polit. Econ.* **98**, 71–102 (1990)
- Segerstrom, P.: Endogenous growth without scale effect. *Am. Econ. Rev.* **88**, 1290–1310 (1998)
- Segerstrom, P., Anant, T., Dinopoulos, E.: A Schumpeterian model of the product life cycle. *Am. Econ. Rev.* **80**, 1077–1091 (1990)
- Shleifer, A.: Implementation cycles. *J. Polit. Econ.* **94**, 1163–1190 (1986)
- Sorger, G.: Horizontal innovations with endogenous quality choice. *Economica* **78**, 697–722 (2011)

- Stein, J.: Waves of creative destruction: firm-specific learning-by-doing and the dynamics of innovation. *Rev. Econ. Stud.* **64**, 265–288 (1997)
- Stokey, N.: Human capital, product quality, and growth. *Q. J. Econ.* **106**, 587–616 (1991)
- Verhoogen, E.: Trade, quality upgrading and wage inequality in the Mexican manufacturing sector. *Q. J. Econ.* **123**, 489–530 (2008)
- Wei, S.-J., Xie, Z., Zhang, X.: From “Made in China” to “Innovated in China”: necessity, prospect, and challenges. *J. Econ. Perspect.* **31**, 49–70 (2017)
- Young, A.: Invention and bounded learning by doing. *J. Polit. Econ.* **101**, 443–472 (1993)
- Zilibotti, F.: Growing and slowing down like China. *J. Eur. Econ. Assoc.* **15**, 943–988 (2017)

**Publisher's Note** Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.