Online Appendix

A Additional theoretical results

A.1 Existence and uniqueness of equilibrium

Proposition 5 *Suppose that, for each commodity* $(z,v) \in \mathbb{Z} \times \mathbb{V}$*,* $\eta_{z,v}$ *is independently drawn from a uniform density function with support* $\left[\underline{\eta}, \overline{\eta}\right]$*. Then, for any* $\kappa > 0$*, in equilibrium:* $w_v = w$ *for all* $v \in \mathbb{V}$*.*

Proof. *Existence of equilibrium:* As a first step, we prove that $w_v = w$ for all $v \in \mathbb{V}$ is an equilibrium of the model. Firstly, notice that when $w_i = w$ for all $i \in \mathbb{V}$, the Lagrange multipliers will be identical for all countries, and in particular we may write $\mu^i = \mu$ for all $i \in \mathbb{V}$. Secondly, using Lemma 1, when $w_v = w$ for all $v \in \mathbb{V}$, conditions in (16) together with (21) and $\mu^i = \mu$ for all $i \in \mathbb{V}$, lead to:

$$q^i_{z,v} = q_{z,v} = \left(\frac{1+\kappa}{Ae^{\eta_{z,v}}\mu}\right)^{1/\left(\eta_{z,v}-1\right)}, \tag{41}$$

$$\beta^i_{z,v} = \beta_{z,v} = \left(\frac{1+\kappa}{A\left(e\mu\right)^{\eta_{z,v}}}\right)^{1/\left(\eta_{z,v}-1\right)}. \tag{42}$$

Now, recall that each $\eta_{z,v}$ is drawn from from an independent uniform probability distribution with support $\left[\underline{\eta}, \overline{\eta}\right]$. Hence, by the law of large numbers, for each country $v \in \mathbb{V}$, the (infinite) sequence of draws $\left\{\eta_{z,v}\right\}_{z \in \mathbb{Z}}$ will also be uniformly distributed over $\left[\underline{\eta}, \overline{\eta}\right]$ along the goods space. This implies that, integrating over $\mathbb{Z}$ and bearing in mind (42), $\int_{\mathbb{Z}} \beta^i_{z,v}\, dz = \int_{\mathbb{Z}} \beta_{z,v}\, dz = \beta_v = \beta > 0$, for each good $v \in \mathbb{V}$. Next, replacing $\int_{\mathbb{Z}} \beta^i_{z,v}\, dz = \beta$ into (19), and swapping the order of integration, we obtain $\int_{\mathbb{V}} \beta\, dv = 1$, which in turn implies that $\beta = 1$ since $\mathbb{V}$ has unit mass. Then, it is easy to check that all conditions (8) hold simultaneously when $w_v = w$ for all $v \in \mathbb{V}$.

Equilibrium uniqueness: We now proceed to prove the above equilibrium is unique. Normalise $w = 1$, and suppose for a subset $\mathcal{J} \subset \mathbb{V}$ of countries with measure $\lambda_j > 0$ we have $w_j > 1$, while for a (disjoint) subset $\mathcal{K} \subset \mathbb{V}$ of countries with measure $\lambda_k > 0$ we have $w_k < 1$. Denote finally by $\mathcal{I} \subset \mathbb{V}$ the (complementary) subset of countries with $w_i = 1$. Consider some $k \in \mathcal{K}$, $i \in \mathcal{I}$, and $j \in \mathcal{J}$, and take $(z_k, k), (z_i, i), (z_j, j)$ such that: $\eta_{z_k,k} = \eta_{z_i,i} = \eta_{z_j,j} = \eta$. Notice that, due to the law of large numbers, for any $\eta \in \left[\underline{\eta}, \overline{\eta}\right]$ the measure of good-variety couples for which the last condition is satisfied is the same in k, i and j.

As a first step, take country $i \in \mathcal{I}$. (16) and (17) imply that, for (z_k, k), (z_i, i) and (z_j, j), we must have, respectively:

$$\begin{aligned}
\ln(1+\kappa)-\ln A &= \eta\ln\left(\mu^i\right)+\ln\left(w_k\right)+(\eta-1)\ln(\beta^i_{z_k,k})+\eta-\delta^i_{z_k,k} \\
&= \eta\ln\left(\mu^i\right)+(\eta-1)\ln(\beta^i_{z_i,i})+\eta-\delta^i_{z_i,i} \\
&= \eta\ln\left(\mu^i\right)+\ln\left(w_j\right)+(\eta-1)\ln(\beta^i_{z_j,j})+\eta-\delta^i_{z_j,j}.
\end{aligned}$$

Notice also from (18) and (21) that if $\delta^i_{z,v}>0$, then $\ln\beta^i_{z,v}=-\ln\mu^i$, whereas if $\delta^i_{z,v}=0$, then $\ln\beta^i_{z,v}\geq-\ln\mu^i$. Then, $\beta^i_{z_k,k}\geq\beta^i_{z_i,i}\geq\beta^i_{z_j,j}$.

As a second step, take country $k\in\mathcal{K}$. (16) and (17) imply that, for (z_k,k), (z_i,i) and (z_j,j), we must have, respectively:

$$\begin{aligned}
\ln(1+\kappa)-\ln A &= \eta\ln(\mu^k)+(\eta-1)\ln(\beta^k_{z_k,k})+\eta-\delta^k_{z_k,k} \\
&= \eta\ln(\mu^k)+\ln\left(\frac{1}{w_k}\right)+(\eta-1)\ln(\beta^k_{z_i,i})+\eta-\delta^k_{z_i,i} \\
&= \eta\ln(\mu^k)+\ln\left(\frac{w_j}{w_k}\right)+(\eta-1)\ln(\beta^k_{z_j,j})+\eta-\delta^k_{z_j,j}.
\end{aligned}$$

Following an analogous reasoning as before, it follows that $\beta^k_{z_k,k}\geq\beta^k_{z_i,i}\geq\beta^k_{z_j,j}$.

As a third step, take country $j\in\mathcal{J}$, and notice $w_j>1$. (16) and (17) imply that, for (z_k,k), (z_i,i) and (z_j,j), we must have, respectively:

$$\begin{aligned}
\ln(1+\kappa)-\ln A &= \eta\ln\left(\mu^j\right)+\ln\left(\frac{w_k}{w_j}\right)+(\eta-1)\ln(\beta^j_{z_k,k})+\eta-\delta^j_{z_k,k} \\
&= \eta\ln\left(\mu^j\right)+\ln\left(\frac{1}{w_j}\right)+(\eta-1)\ln(\beta^j_{z_i,i})+\eta-\delta^j_{z_i,k} \\
&= \eta\ln\left(\mu^j\right)+(\eta-1)\ln(\beta^j_{z_j,j})+\eta-\delta^j_{z_j,j}.
\end{aligned}$$

Again, an analogous reasoning as in the previous cases leads to $\beta^j_{z_k,k}\geq\beta^j_{z_i,i}\geq\beta^j_{z_j,j}$.

Finally, integrate among the good space $\mathbb{Z}$ and country space $\mathbb{V}$. The above results lead to:

$$\begin{aligned}
&\lambda^j w_j\int_{\mathbb{Z}}\beta^j_{z,k}dz+\lambda^k w_k\int_{\mathbb{Z}}\beta^k_{z,k}dz+\left(1-\lambda^j-\lambda^k\right)\int_{\mathbb{Z}}\beta^i_{z,k}dz\geq \\
&\lambda^j w_j\int_{\mathbb{Z}}\beta^j_{z,i}dz+\lambda^k w_k\int_{\mathbb{Z}}\beta^k_{z,i}dz+\left(1-\lambda^j-\lambda^k\right)\int_{\mathbb{Z}}\beta^i_{z,i}dz\geq \\
&\lambda^j w_j\int_{\mathbb{Z}}\beta^j_{z,j}dz+\lambda^k w_k\int_{\mathbb{Z}}\beta^k_{z,j}dz+\left(1-\lambda^j-\lambda^k\right)\int_{\mathbb{Z}}\beta^i_{z,j}dz.
\end{aligned}\tag{43}$$

Note that the first line in (43) equals the world spending on commodities produced in k, the second equals the world spending on commodities produced in i, and the third equals the world spending on commodities produced in j. However, when $w_k<1<w_j$, those inequalities are inconsistent with market clearing conditions (8). As a result, there cannot exist an equilibrium with measure $\lambda_j>0$ of countries with $w_j>1$ and/or a measure $\lambda_k>0$ of countries with $w_k<1$. ■

Proposition 6 *Suppose that the set $\mathbb{V}$ is composed by two disjoint subsets with positive measure: H and L. Assume that: a) for any $(z,h) \in \mathbb{Z} \times H$, $\eta_{z,h}$ is independently drawn uniform density function with support $[\underline{\eta}, \overline{\eta}]$; b) for any $(z,l) \in \mathbb{Z} \times L$, $\eta_{z,l} = \overline{\eta}$. Then, for any $h, h', h'' \in H$ and $l, l', l'' \in L$: (i) $w_{h'} = w_{h''}$; (ii) $w_{l'} = w_{l''}$; (iii) $w_h > w_l$.*

Proof. We prove the proposition in different steps. We first prove that, if an equilibrium exists, then it must necessarily be the case that, for any $h, h', h'' \in H$ and $l, l', l'' \in L$: 1) $w_h \neq w_l$; 2) $w_{h'} = w_{h''}$ and $w_{l'} = w_{l''}$; 3) $w_h/w_l > 1$; 4) $w_h/w_l < \infty$. Lastly, we prove that a unique equilibrium exists, with: 5) $1 < w_h/w_l < \infty$.

Preliminarily, consider a generic country $i \in \mathbb{V}$, and compute the aggregate demand by i for goods produced in country $v \in \mathbb{V}$. From the first-order conditions, it follows that:

$$\beta^i_{z,v} = \max\left\{ \left[\frac{(1+\kappa)(w_i/w_v)}{A(e\mu^i)^{\eta_{z,v}}}\right]^{\frac{1}{\eta_{z,v}-1}}, \frac{1}{\mu^i} \right\}. \tag{44}$$

Hence, total demand by i for goods produced in $h \in H$ and in $l \in L$ are respectively given by:

$$\int_{\mathbb{Z}} \beta^i_{z,h}\, w_i\, dz = w_i \int_{\underline{\eta}}^{\overline{\eta}} \max\left\{ \left(\frac{1+\kappa}{A(e\mu^i)^{\eta}} \frac{w_i}{w_h}\right)^{1/(\eta-1)}, \frac{1}{\mu^i} \right\} \frac{1}{\overline{\eta}-\underline{\eta}} d\eta, \quad \text{for any } h \in H, \tag{45}$$

and

$$\int_{\mathbb{Z}} \beta^i_{z,l}\, w_i\, dz = w_i \max\left\{ \left(\frac{1+\kappa}{A(e\mu^i)^{\overline{\eta}}} \frac{w_i}{w_l}\right)^{1/(\overline{\eta}-1)}, \frac{1}{\mu^i} \right\}, \quad \text{for any } l \in L. \tag{46}$$

Step 1. Suppose now that, in equilibrium, $w_i = w$ for all $i \in \mathbb{V}$. Recalling the proof of Lemma 1, we can observe that the constraints $q^i_{z,v} \geq 1$ will not bind in this case. Demand intensities in (44) are then given by $\beta^i_{z,v} = \beta_{z,v} = (e \cdot \mu)^{-\eta_{z,v}/(\eta_{z,v}-1)} [(1+\kappa)/A]^{1/(\eta_{z,v}-1)}$ for all $i \in \mathbb{V}$. As a result, the value in (45) must be strictly larger than the value in (46), since the term $[(1+\kappa)/A]^{1/(\eta-1)} / \mu^{\eta/(\eta-1)}$ is strictly decreasing in η. As a consequence, given that i represents a generic country in $\mathbb{V}$, integrating over the set $\mathbb{V}$, it follows that the world demand for goods produced in a country from H will be strictly larger than the world demand for goods produced in a country from L. But this is inconsistent with the market clearing conditions, which require that world demand is equal for all $v \in \mathbb{V}$. Hence, $w_v = w$ for all $v \in \mathbb{V}$ cannot hold in equilibrium.

Step 2. Suppose that, in equilibrium, $w_{h'} > w_{h''}$ for some $h', h'' \in H$. Computing (45) respectively for h' and h'' yields:

$$w_i \int_{\underline{\eta}}^{\overline{\eta}} \max\left\{ \left(\frac{1+\kappa}{A(e\mu^i)^{\eta}} \frac{w_i}{w_{h'}}\right)^{\frac{1}{\eta-1}}, \frac{1}{\mu^i} \right\} \frac{1}{\overline{\eta}-\underline{\eta}} d\eta \leq w_i \int_{\underline{\eta}}^{\overline{\eta}} \max\left\{ \left(\frac{1+\kappa}{A(e\mu^i)^{\eta}} \frac{w_i}{w_{h''}}\right)^{\frac{1}{\eta-1}}, \frac{1}{\mu^i} \right\} \frac{1}{\overline{\eta}-\underline{\eta}} d\eta$$

Now, since i represents a generic country in $\mathbb{V}$, integrating over the set $\mathbb{V}$, it follows that the world demand for goods produced in country h' will be no larger than the world demand for goods produced in country h''. But this is inconsistent with the market clearing conditions, which require that world demand for goods produced in country h' must be strictly larger than world demand for goods produced in country h''. Furthermore, an analogous reasoning rules out $w_{h'} < w_{h''}$. As a consequence, it must be the case that, if an equilibrium exists, it must be characterised by $w_{h'} = w_{h''}$ for any $h', h'' \in H$. (Similarly, it can be proved that, if an equilibrium exists, it must be characterised by $w_{l'} = w_{l''}$ for any $l', l'' \in L$.)

Step 3. Suppose that $w_h < w_l$. Since $\left\{\left[(1+\kappa)/A\right](w_i/w_v)/\left(\mu^i\right)^\eta\right\}^{1/(\eta-1)}$ is strictly decreasing in η, it follows that the value in (46) is no larger than the value in (45). Moreover, since i represents a generic country in $\mathbb{V}$, integrating over the set $\mathbb{V}$, we obtain that the world demand for goods produced in a country from region L is no larger than world demand for goods produced in a country from region H. But this is inconsistent with the market clearing conditions when $w_h < w_l$, which require that world demand for goods produced in a country from region L must be strictly larger than world demand for goods produced in a country from region H.

Step 4. As a result of steps 1, 2 and 3, our only remaining candidate for an equilibrium is then $w_h > w_l$. From (45), it follows that the aggregate demand by any $h' \in H$ for goods produced in region H coincides with its aggregate supply to the same region. Hence, there must be no net surplus within region H. Analogously, from (46) it follows that there must be no net surplus within region L. As a result, a necessary condition for market clearing is that the aggregate demand by region L for goods produced in region H must equal the aggregate demand by region H for goods produced in region L. Formally:

$$\int_L \int_H \int_{\mathbb{Z}} \beta_{z,h}^{l'} \, w_{l'} \, dz \, dh \, dl' = \int_H \int_L \int_{\mathbb{Z}} \beta_{z,l}^{h'} \, w_{h'} \, dz \, dl \, dh' \tag{47}$$

Suppose now that $w_h \to \infty$. Then, on the one hand, from (45) we obtain the aggregate demand by $l' \in L$ for goods produced in region H would be equal to a finite (non-negative) number. Since this would hold true for every $l' \in L$, then the aggregate demand by region L for goods produced in region H —left-hand side of (47)— would be equal to a finite (non-negative) number. On the other hand, from (45) it follows that when $w_h \to \infty$ the aggregate demand by $h' \in H$ for goods produced in any $l \in L$ would tend to infinity. Since this would hold true for every $h' \in H$ and $l \in L$, then the aggregate demand by region H for goods produced in region L —right-hand side of (47)— would also tend to infinity. But this then is inconsistent with the equality required by condition (47). Hence, if an equilibrium exists, it must be then characterised by $w_l < w_h < \infty$.

Step 5. Finally, we prove now that there exists an equilibrium $1 < w_h/w_l < \infty$, and this equilibrium is unique. Recall that w_h/w_l represents the relative wage between region H and region L. Step 1 shows that, should the relative wage equal one, then the world demand for goods produced in a country from H would be strictly larger than the world demand for goods produced in a country from L. Step 4 shows instead that, should $w_h \to \infty$, then the world demand for goods produced in a country from H would be strictly smaller than the world demand for goods produced in a country from L. Consider now (44) for any $v = h$, and notice that the demand intensities $\beta^i_{z,h}$ are all non-increasing in w_h/w_l. In addition, consider (44) for any $v = l$, and notice that in this case the $\beta^i_{z,l}$ are all non-decreasing in w_h/w_l, while they are strictly increasing in w_h/w_l for at least some $z \in \mathbb{Z}$ when $i \in H$. Therefore, taking all this into account, together with the expressions in (45) and (46), it follows that the world demand for goods produced in a country from L may increase with w_h/w_l, while world demand for goods produced in a country from H will decrease with w_h/w_l. Hence, by continuity, there must necessarily exist some $1 < w_h/w_l < \infty$ consistent with all market clearing conditions holding simultaneously. In addition, this equilibrium must then also be unique. ■

A.2 Formalisation of results discussed in Appendix B

Proposition 7 *Suppose that the set $\mathbb{V}$ is composed by K disjoint subsets, indexed by $k = 1, ..., K$, each denoted by $\mathcal{V}_k \subset \mathbb{V}$ and with Lebesgue measure $\lambda_k > 0$. Assume that for any country $v_k \in \mathcal{V}_k$ each η_{z,v_k} is independently drawn from a uniform distribution with support $[\eta_k, \overline{\eta}]$, with $\eta_{k'} < \eta_{k''}$ for $k' < k''$. Then: $w_1 > ... > w_{k'} > ... > w_K$, where $1 < k' < K$.*

Proof. Combining (16) and (17), yields:

$$\beta^i_{z,v} = \max\left\{ \left[\left(\frac{1+\kappa}{A}\right) \left(\frac{w_i}{w_v}\right) \left(e \cdot \mu^i\right)^{-\eta_{z,v}} \right]^{1/(\eta_{z,v}-1)}, \frac{1}{\mu^i} \right\} \equiv \beta^i\left(\eta_{z,v}, w_v\right). \tag{48}$$

Notice from (48) that $\partial\beta^i\left(\eta_{z,v}, w_v\right)/\partial\eta_{z,v} \leq 0$ and $\partial\beta^i\left(\eta_{z,v}, w_v\right)/\partial w_v \leq 0$.

Consider now two generic regions $k' < k''$, and suppose that $w_{k'} \leq w_{k''}$. Since the distribution of $\eta_{z,k'}$ FOSD the distribution of $\eta_{z,k''}$, then it follows that $\int_{\mathbb{Z}} \beta^i_{z,k'} dz \geq \int_{\mathbb{Z}} \beta^i_{z,k''} dz$. Moreover, recalling the proof of Lemma 1 it follows that the $\beta^i_{z,v}$ in (48) must be *strictly* decreasing in $\eta_{z,v}$ and in w_v at least in one of all the regions in the world.[28] As a result, there will exist a positive measure of countries for which $\int_{\mathbb{Z}} \beta^i_{z,k'} dz > \int_{\mathbb{Z}} \beta^i_{z,k''} dz$ when $w_{k'} \leq w_{k''}$. Therefore, integrating

[28] More precisely, it must be that the $\beta^i_{z,v}$ in (48) are strictly decreasing in $\eta_{z,v}$ and w_v *at least* in region k^*, such that $w_{k^*} \in \max\{w_1, ..., w_K\}$. That is, the region (or regions) exhibiting with the highest wage.

over the set $\mathbb{V}$, we obtain that $\int_{\mathbb{V}}\int_{\mathbb{Z}}\beta^{i}_{z,k'}dz > \int_{\mathbb{V}}\int_{\mathbb{Z}}\beta^{i}_{z,k''}dz$. That is, the world demand for goods produced in a country from region k' is strictly larger than world demand for goods produced in a country from region k''. But this is inconsistent with the market clearing conditions when $w_{k'} \leq w_{k''}$, which require that world demand for goods produced in a country from region k' must be no larger than world demand for goods produced in a country from region k''. As a consequence, it must be that $w_{k'} > w_{k''}$. ■

Proposition 8 *For country $v_1 \in \mathcal{V}_1$ such that $\eta_{z,v_1} = \underline{\eta}$ and any country $v_k \in \mathcal{V}_k$ such that $\eta_{z,v_k} = \eta_k$ and $k \neq 1$: $RCA_{z,v_1} > RCA_{z,v_k}$, for any $z \in \mathbb{Z}$.*

Proof. Countries with identical incomes have identical budget shares. Let $\beta^{j}_{z,v}$ denote the common budget share for (z,v) in j. Then, from the definition of total production of good z by country v, we have that $X_{z,v} = \sum_{j=1}^{K}\lambda_j\beta^j\left(\eta_{z,v},w_v\right)w_j$. Notice also that $X_v = w_v$ and $W_z/W = 1$. Hence, (10) yields:

$$RCA_{z,v} = \frac{\sum_{j=1}^{K}\lambda_j\beta^j\left(\eta_{z,v},w_v\right)w_j}{w_v}. \tag{49}$$

Consider a generic good $z \in \mathbb{Z}$ and, without loss of generality, select countries: $v_1 \in \mathcal{V}_1$ such that $\eta_{z,v_1} = \underline{\eta}$; and $v_k \in \mathcal{V}_k$ from any region $k \in (1,K]$ such that $\eta_{z,v_k} = \eta_k$. From (49) we obtain that $RCA_{z,v_1} > RCA_{z,v_k}$ requires:

$$\frac{\sum_{j=1}^{K}\lambda_j\beta^j\left(\underline{\eta},w_1\right)w_j}{w_1} > \frac{\sum_{j=1}^{K}\lambda_j\beta^j\left(\eta_k,w_k\right)w_j}{w_k}. \tag{50}$$

Notice too that market clearing conditions imply:

$$\int_{\mathbb{Z}}\left[\sum_{j=1}^{K}\lambda_j\beta^j\left(\eta_{z,1},w_1\right)w_j\right]dz = w_1 \quad \text{and} \quad \int_{\mathbb{Z}}\left[\sum_{j=1}^{K}\lambda_j\beta^j\left(\eta_{z,k},w_k\right)w_j\right]dz = w_k.$$

Therefore, it follows that $\int_{\mathbb{Z}} RCA_{z,v_1}dz = \int_{\mathbb{Z}} RCA_{z,v_k}dz = 1$. We can transform the integrals over z in integrals over η, to obtain:

$$\frac{1}{\overline{\eta}-\underline{\eta}}\int_{\underline{\eta}}^{\overline{\eta}}\left[RCA_{\eta,v_1}\right]d\eta = 1, \tag{51}$$

$$\frac{1}{\overline{\eta}-\eta_k}\int_{\eta_k}^{\overline{\eta}}\left[RCA_{\eta,v_k}\right]d\eta = 1 \tag{52}$$

Recall that $\partial\beta^j\left(\cdot\right)/\partial\eta < 0$, implying that $\partial\left(RCA_{\eta,v}\right)/\partial\eta < 0$. Moreover, since $w_k < w_1$, notice that it must be the case that $RCA_{\eta,v_k} > RCA_{\eta,v_1}$ for any $\eta \in [\eta_k, \overline{\eta}]$. Now, suppose that

$RCA_{\eta_k,v_k} \geq RCA_{\underline{\eta},v_1}$, then bearing in mind that $\partial^2\beta^j(\cdot)/(\partial\eta)^2 > 0$ and $\partial^2\beta^j(\cdot)/(\partial\eta\partial w_v) > 0$, we can observe that when (52) holds true then

$$\frac{1}{\overline{\eta}-\underline{\eta}}\int_{\underline{\eta}}^{\overline{\eta}} [RCA_{\eta,v_1}]\, d\eta < 1,$$

which contradicts (51). Therefore, it must be the case that $RCA_{\eta_k,v_k} < RCA_{\underline{\eta},v_1}$. ■

Proposition 9 *Let $\beta^j_{z,\underline{\eta}}$ denote the demand intensity by a consumer from region $j \in \mathcal{V}_j$ for the variety of good z produced in country v_1 such that $\eta_{z,v_1} = \underline{\eta}$. Then: $\beta^1_{z,\underline{\eta}} > ... > \beta^{j'}_{z,\underline{\eta}} > ... > \beta^K_{z,\underline{\eta}}$, where $1 < j' < K$.*

Proof. Consider a pair of generic consumers from regions j' and j'', where $j' < j''$. In addition, consider a pair of generic exporters from countries $v_{k'}$ and $v_{k''}$, where $k' \leq k''$. Following an analogous procedure as in the proof of Proposition 4, combining (16) and (17) of consumers j' and j'' for the varieties of good z produced in $v_{k'}$ and $v_{k''}$, we may obtain:

$$\begin{aligned}\left(\eta_{z,v_{k''}} - \eta_{z,v_{k'}}\right)\ln\left(\mu^{j'}/\mu^{j''}\right) + \left(\delta^{j''}_{z,v_{k'}} - \delta^{j'}_{z,v_{k'}}\right) + \left(\delta^{j''}_{z,v_{k''}} - \delta^{j'}_{z,v_{k''}}\right) = \\ \left(\eta_{z,v_{k'}} - 1\right)\ln\left(\beta^{j'}_{z,v_{k'}}/\beta^{j''}_{z,v_{k'}}\right) + \left(\eta_{z,v_{k''}} - 1\right)\ln\left(\beta^{j''}_{z,v_{k''}}/\beta^{j'}_{z,v_{k''}}\right).\end{aligned} \tag{53}$$

Since $\ln\left(\mu^{j'}/\mu^{j''}\right) > 0$ and $\delta^{j''}_{z,v_k} \geq \delta^{j'}_{z,v_k}$, from (53) it follows that $\beta^{j'}_{z,v_{k'}}/\beta^{j''}_{z,v_{k'}} > \beta^{j'}_{z,v_{k''}}/\beta^{j''}_{z,v_{k''}}$ when $\eta_{z,v_{k'}} < \eta_{z,v_{k''}}$. Now, let $k' = 1$ and pick z such that $\eta_{z,v_1} = \underline{\eta}$. Next, suppose $\beta^{j'}_{z,\underline{\eta}} \leq \beta^{j''}_{z,\underline{\eta}}$. Then, we must have that $\beta^{j'}_{z,v} \leq \beta^{j''}_{z,v}$ for all $(z,v) \in \mathbb{Z}\times\mathbb{V}$, with strict inequality for all (z,v) such that $\eta_{z,v_k} > \underline{\eta}$. However, since the budget constraints of consumer j' and j'' require that $\int_{\mathbb{Z}}\int_{\mathbb{V}}\beta^{j'}_{z,v}\, dv\, dz = \int_{\mathbb{Z}}\int_{\mathbb{V}}\beta^{j''}_{z,v}\, dv\, dz$, then $\beta^{j'}_{z,\underline{\eta}} \leq \beta^{j''}_{z,\underline{\eta}}$ cannot possibly be true. ■

A.3 Trade frictions and consumer loss

Consider country i as an importer of good z. We assume this commodity is subject to a tariff $t_z > 0$ applied on the (free-on-board) price of imports, regardless of the quality level in which it is imported by i. Since the tariff is applied only to one (atomless) sector within a continuum of sectors, we can disregard general equilibrium effects as they would be negligible. Given the tariff t_z, the final price at which good z sourced from a generic country $v \neq i$ will be sold to consumers in country i will be:

$$p^i_{z,v}(q) = (1+t_z)\, A q^{\eta_{z,v}} w_v/(1+\kappa). \tag{54}$$

Recall the utility function of the individual (3), and focus on the sub-utility derived from the consumption of good z sourced from country v. Let us write this sub-utility as $u_{z,v} = \ln(c_{z,v})^{q_{z,v}}$.

Bearing in mind (54), and considering that, in the optimum:

$$q_{z,v}^{i}\left(t_{z}\right)=\left(\frac{1+\kappa}{1+t_{z}}\frac{1}{AQ_{i}e^{\eta_{z,v}}}\frac{w_{i}}{w_{v}}\right)^{1/\left(\eta_{z,v}-1\right)}, \tag{55}$$

and $\beta_{z,v}^{i}=q_{z,v}^{i}/Q_{i}$, then $u_{z,v}$ boils down to:

$$u_{z,v}=\eta_{z,v}\left(\frac{1+\kappa}{1+t_{z}}\frac{1}{AQ_{i}e^{\eta_{z,v}}}\frac{w_{i}}{w_{v}}\right)^{\frac{1}{\eta_{z,v}-1}}. \tag{56}$$

Let us denote by $\Gamma_{v}^{i}\left(t_{z}\right)$ the utility loss due to imposing an import tariff $t_{z}>0$ relative to the case where $t_{z}=0$. From (56), we get:

$$\Gamma_{v}^{i}\left(t_{z}\right)=\eta_{z,v}\left(\frac{1+\kappa}{AQ_{i}e^{\eta_{z,v}}}\frac{w_{i}}{w_{v}}\right)^{\frac{1}{\eta_{z,v}-1}}\left[1-\left(1+t_{z}\right)^{-\frac{1}{\eta_{z,v}-1}}\right]. \tag{57}$$

It is plain from (57) that $\Gamma_{v}^{i}(t_{z})>0$ whenever $t_{z}>0$, and that $\partial\Gamma_{v}^{i}\left(\cdot\right)/\partial t_{z}>0$.

More interesting is studying how the tariff loss function behaves at different levels of income of the importer. Using (57), we may compute the elasticity of $\Gamma_{v}^{i}(t_{z})$ with respect to w_{i}, to obtain:

$$\frac{\partial\ln\Gamma_{v}^{i}(t_{z})}{\partial\ln w_{i}}=\frac{1}{\eta_{z,v}-1}>0. \tag{58}$$

This result implies that the utility loss due to the tariff is greater for richer consumers. Moreover, this difference in welfare loss between richer and poorer importers becomes greater when the tariff is imposed on more efficient producers of good z (i.e., when the tariff is imposed on countries that received a lower $\eta_{z,v}$).

Our model then yields the following two qualitative welfare loss results. First, the consumer loss due to import tariffs is always greater for richer importers. Second, the loss disparity between richer and poorer importers gets larger when the tariff is imposed on more efficient producers of good z. These two results crucially rest on our nonhomothetic preference structure (in Section A.5 below we show that these two results vanish away in the presence of homothetic preferences).

Lastly, we can use some of the above expressions to get a sense of the relative magnitude of welfare loss due to the tariff implied by our model. Bearing in mind (58), from (55) we can observe that:

$$\frac{\partial\ln q_{z,v}^{i}}{\partial\ln w_{i}}=\frac{1}{\eta_{z,v}-1}=\frac{\partial\ln\Gamma_{v}^{i}(t_{z})}{\partial\ln w_{i}}. \tag{59}$$

Interestingly, the magnitude of the left-hand side of this expression can be obtained from the data by using unit values as a proxy for $q_{z,v}^{i}$. The pooled estimation of the log of unit values on the log of importer's income delivers a mean value of 0.075.[29] This implies that a 10% richer importer

[29] We conduct our pooled regression using the data on value of imports and quantity of imports by product at the 6-digit Harmonised System (HS-6) level of disaggregation in year 2009. Our regression includes a full set of product-exporter dummies. Full details of this regression are available from the authors upon request.

will suffer a 0.75% higher welfare loss when importing good z from the average producer.[30] If we add (subtract) one standard deviation to the estimate, the welfare loss suffered by a 10% richer importer rises (declines) to 0.85% (0.65%).[31]

A.4 Sectoral subsidy and comparative advantage

Consider country v as a producer of good z, and assume that local producers of good z receive a proportional subsidy $\sigma_{z,v}$ regardless of the quality level of their output. Since the subsidy is applied only to one (atomless) sector within a continuum of sectors, we can disregard again general equilibrium effects as they would be negligible. Following an analogous reasoning as before, they will sell their output to consumers of country i at price $p^i_{z,v}(q) = (1-\sigma_{z,v})\, Aq^{\eta_{z,v}} w_v / (1+\kappa)$. This, in turn, implies that the share of income of consumers in country i spent on good z produced in country v is given by:[32]

$$\beta^i_{z,v} = \left[\frac{(1+\kappa)}{(1-\sigma_{z,v})\, A\, (e\mu^i)^{\eta_{z,v}}} \frac{w_i}{w_v}\right]^{\frac{1}{\eta_{z,v}-1}}. \tag{60}$$

Let $S_{z,v}$ denote the share of sector z in the total GDP of country v. Bearing in mind that, in this simplified version of the model, there are two countries in region L (with income w_l) and two countries in region H (with income w_h), it follows that:

$$S_{z,v} = \frac{2}{w_v}\left(w_h \beta^H_{z,v} + w_l \beta^L_{z,v}\right). \tag{61}$$

Differentiating (60) with respect to $\sigma_{z,v}$ yields:

$$\frac{d\beta^i_{z,v}}{d\sigma_{z,v}} = \frac{\beta^i_{z,v}}{\left(\eta_{z,v}-1\right)(1-\sigma_{z,v})} > 0. \tag{62}$$

The impact of a subsidy to sector z in country v on its the GDP share can be obtained by differentiating (61) with respect to $\sigma_{z,v}$ while bearing in mind (62). Thus,

$$\frac{dS_{z,v}}{d\sigma_{z,v}} = \frac{2}{w_v}\left(w_h \frac{d\beta^H_{z,v}}{d\sigma_{z,v}} + w_l \frac{d\beta^L_{z,v}}{d\sigma_{z,v}}\right). \tag{63}$$

[30] Using (59), we can back out the implied value of the sectoral productivity draw for the average producer, $\hat{\eta} = 14.33$. Performing a similar pooled regression analysis as the one we do here, but including a panel of transactions instead of data for year 2009 only, Fieler (2012, Table 2) obtains an estimate equal to 0.06. According to her estimate (which is just slightly smaller in magnitude than ours), the sectoral productivity of average producer would be $\hat{\eta} = 17.67$.

[31] Again, using (59), the implied value of the sectoral productivity draw for the more (less) productive exporter when we add (subtract) one standard deviation to the estimated value is $\hat{\eta} = 12.76$ ($\hat{\eta} = 16.38$). Repeating this exercise for a two standard deviations difference, we obtain a 1.05% (0.45%) higher welfare loss, corresponding to sectoral a sectoral productivity draw for the exporter of $\hat{\eta} = 10.52$ ($\hat{\eta} = 23.22$)

[32] Notice that when $i = v$ then $\beta^i_{z,v}$ refers to domestic sales of good z.

Let us now compare the impact of the subsidy $\sigma_{z,v}$ on the sectoral share $S_{z,v}$ for the case of a country in region L (which, by construction, must have received $\overline{\eta}$ as productivity draw in sector z) and the country in region H that received $\overline{\eta}$ as productivity draw in sector z. That is, we are computing the derivative (63) for two economies with different wages in the denominator (w_l and w_h, respectively), but both sharing the same elasticity of quality upgrading $\eta_{z,v} = \overline{\eta}$. Notice now that both $d\beta^H_{z,v}/d\sigma_{z,v}$ and $d\beta^L_{z,v}/d\sigma_{z,v}$ are always larger in a country from region L than in the country from region H that received the productivity draw $\eta_{z,v} = \overline{\eta}$. Therefore, the effect of $\sigma_{z,v}$ on $S_{z,v}$ will be larger in the country from region L than in the country from region H that received the productivity draw $\eta_{z,v} = \overline{\eta}$. This uneven effect of the subsidy across producers with different incomes rests crucially on our non-homothetic preference structure, as it is shown next in Section A.5.

A.5 Homothetic preferences

We now introduce an alternative preference specification, designed to deliver homothetic demand schedules. For the remaining of this appendix, to streamline the illustration it proves convenient to exploit the ordinal nature of the quality ladders and apply the following monotonic transformation to the quality index: $\tilde{q}_{z,v} = \ln q_{z,v}$. This transformation comes at no loss of generality since the result derived here would obtain even without such transformation.[33]

Suppose that preferences, while retaining the same structure across goods, are for each good now represented by the sub-utility index:

$$u_{z,v} = \ln(\tilde{q}_{z,v} c_{z,v}).$$

This index replaces the expression $\ln (c_{z,v})^{q_{z,v}}$ in (3). The rest of the model remains unchanged. Individuals choose the optimal values of quality and consumption to maximise that utility function subject to (4). There, the pricing function in terms of $\tilde{q}_{z,v}$ becomes:

$$p_{z,v}(\tilde{q}_{z,v}) = \frac{A\, w_v}{1+\kappa} (q_{z,v})^{\eta_{z,v}} = \frac{A\, w_v}{1+\kappa} \left(e^{\tilde{q}_{z,v}}\right)^{\eta_{z,v}} = \frac{A\, w_v}{1+\kappa} e^{\eta_{z,v}\tilde{q}_{z,v}}. \tag{64}$$

Following an analogous reasoning as the one used in Appendix A to solve the original consumer i optimisation problem, we obtain the the (relevant) first-order conditions:

$$\frac{1}{\tilde{q}^i_{z,v}} - \eta_{z,v} = 0, \tag{65}$$

$$\frac{1}{\Omega \cdot \Lambda} \frac{1}{\beta^i_{z,v}} - v^i = 0. \tag{66}$$

[33] More precisely, all the homothetic results we show below will remain qualitatively unchanged if we keep $\tilde{q} = q$. These additional results are available from the authors upon request.

Note that (66) implies that budget shares are identical for all goods, wherever produced. Recalling that budget shares must sum up to one, from (65) and (66) we can thus obtain the following two expressions respectively identifying, for each good z and country v, the optimal quality level and budget share:

$$\tilde{q}^i_{z,v} = \frac{1}{\eta_{z,v}}; \quad \text{and} \quad \beta^i_{z,v} = 1.$$

In the light of these findings, we can show that the results discussed in Appendices A.3 and A.4 vanish away once we modify the utility function to deliver homothetic preferences.

First, consider again (54), now expressed in terms of $\tilde{q}^i_{z,v}$:

$$p_{z,v}\left(\tilde{q}_{z,v}\right) = \left(1 + t_z\right) A e^{\tilde{q}_{z,v}\eta_{z,v}} w_v / \left(1 + \kappa\right). \tag{67}$$

Using the sub-utility index $u_{z,v} = \ln(\tilde{q}^i_{z,v} c^i_{z,v})$, replacing $c^i_{z,v} = \beta^i_{z,v} w_i / p_{z,v}\left(\tilde{q}_{z,v}\right)$ and then $p_{z,v}\left(\tilde{q}_{z,v}\right)$ by (67) and $\tilde{q}_{z,v}$ and $\beta^i_{z,v}$ by their optimal values, we obtain the welfer loss function:

$$u_{z,v} = -\ln \eta_{z,v} + \ln\left(\frac{1+\kappa}{1+t_z}\frac{1}{eA}\frac{w_i}{w_v}\right).$$

Differentiating with respect to t_z yields $\partial u_{z,v}/\partial t_z = -\left(1 + t_z\right)^{-1} < 0$, from which it is easy to observe that the value of the derivative in (58) in this case equals zero.[34] This implies that the utility loss due to the tariff is now independent of consumers' income. In other words, differently from our model with nonhomothetic preferences, in the presence of homothetic preferences (i.e., when willingness to pay for quality is constant), the utility loss due to the import tariff is the same for all individuals, regardless their income level.

Second, recalling the definition of $S_{z,v}$ in (61), from the fact that $\beta_{z,v} = 1$ it straightforwardly follows that the derivatives in (62), and therefore in (63), are in this case equal to zero. This implies that the effect of a subsidy on the share of sector z in the total GDP is now independent of the region to which country v belongs.

[34] Note that the welfare loss function is this case reads $\Gamma^i_v\left(t_z\right) = \ln\left(1 + t_z\right)$, from which $\partial\Gamma^i_v\left(t_z\right)/\partial w_i = 0$ straightforwardly obtains.