Online Appendix to the paper
Quality Ladders in a Ricardian Model of Trade with Nonhomothetic Preferences

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August 2010

Appendix D Auxiliary derivations and additional material

D.1 Auxiliary Derivations

Lemma D1. Let $\underline{\kappa} \equiv a(0)\exp[\eta(0)]$. Then:

(i) For all $\kappa \in (0, \underline{\kappa})$: $\mathbb{L} = \emptyset$;

(ii) for all $\kappa \geq \underline{\kappa}$: $\mathbb{L} = [0, \tilde{v}(\kappa)]$; where $\tilde{v}(\kappa) : [\underline{\kappa}, \infty) \to [0,1]$; $\tilde{v}(\underline{\kappa}) = 0$; $\tilde{v}'(\kappa) \geq 0$, with strict inequality if $\tilde{v}(\kappa) < 1$.

Proof. Part (i). When $\kappa \in (0, \underline{\kappa})$, conditions stipulated in (20) and (22) applied on $v = 0$ entail that: $q_0 = 1$ and $\lambda_0 > 0$. As a result, from Lemma 1 it follows that $q_v = 1$, $\forall v \in \mathbb{V}$. Therefore, since $a'(v) \geq 0$ and $\eta'(v) > 0$, again from (22), $\lambda_v > 0$ for all $v \in \mathbb{V}$ obtains, and thus $\mathbb{L} = \emptyset$.

Part (ii). Firstly, note that (22) applied on $v = 0$, in conjunction Lemma 1, implies that when $\kappa = \underline{\kappa}$, then $\lambda_0 = 0$ and $q_0 = 1$. Then, Lemma 1 implies $Q = 1$. Using these results in (22) yields:

$$\lambda_v = \eta(v) + \ln[\alpha(v)/w] - \ln \underline{\kappa},$$

implying that $\lambda_v > 0$ for all $v \in (0, 1]$. As a result, the set $\mathbb{L} = \emptyset$, meaning that $\tilde{v}(\underline{\kappa}) = 0$. Secondly, notice that, from Lemma D.2 below, $\partial q_v(v)/\partial v < 0$ when $q_v > 1$, hence the set $\mathbb{L} \subseteq \mathbb{V}$ comprises the lower-indexed goods in $\mathbb{V}$, with $\tilde{v}(\underline{\kappa})$ representing its upper bound. Given Lemma 1 and Lemma D.3 below, the aggregate quality index can be written as follows:

$$Q = 1 - \tilde{v}(\kappa) + \int_0^{\tilde{v}(\kappa)} q_v \, dv.$$

Furthermore, observe that, whenever $\tilde{v}(\kappa) < 1$, $\ln(\kappa/Q) = \eta(\tilde{v}(\kappa)) + \ln[\alpha(\tilde{v}(\kappa))/w]$ must hold in equilibrium. This last condition yields, after some simple algebra:

$$Q = \kappa w \exp[-\eta(\tilde{v})]/\alpha(\tilde{v}).$$

In addition to that, because of Lemma 1, in equilibrium:

$$[\eta(v) - 1]\ln q_v = \ln(\kappa/Q) - \eta(v) - \ln[\alpha(v)/w]$$

must hold for any $v \leq \tilde{v}(\kappa)$. Using the former in the latter, we may obtain:

$$q_v = q_v(\tilde{v}(\kappa)) \equiv \left[\frac{\alpha(\tilde{v}(\kappa))}{\alpha(v)}\right]^{\frac{1}{\eta(v)-1}} \exp\left[\frac{\eta(\tilde{v}(\kappa)) - \eta(v)}{\eta(v) - 1}\right], \ \forall v \in [0, \tilde{v}(\kappa)]. \quad (29)$$

In equilibrium, it must be the case that:

$$\kappa w \exp\left[-\eta\left(\tilde{v}(\kappa)\right)\right]/a\left(\tilde{v}(\kappa)\right) = 1 - \tilde{v}(\kappa) + \int_0^{\tilde{v}(\kappa)} q_v\left(\tilde{v}(\kappa)\right) dv, \tag{30}$$

where the right hand-side of (30) uses (29). Computing the total differentiation of (30), yields after some algebra:[32]

$$\frac{Q}{\kappa}d\kappa = \left[\frac{\alpha'\left(\tilde{v}(\kappa)\right)}{\alpha\left(\tilde{v}(\kappa)\right)} + \eta'\left(\tilde{v}(\kappa)\right)\right]\left[Q + \int_0^{\tilde{v}(\kappa)} \frac{q_v}{\eta(v)-1}dv\right] d\tilde{v},$$

leading finally to:

$$\frac{d\tilde{v}}{d\kappa} = \left[\frac{\kappa}{Q}\left(\frac{\alpha'\left(\tilde{v}(\kappa)\right)}{\alpha\left(\tilde{v}(\kappa)\right)} + \eta'\left(\tilde{v}(\kappa)\right)\right)\left(1 - \tilde{v}(\kappa) + \int_0^{\tilde{v}(\kappa)} \frac{\eta(v)}{\eta(v)-1}q_v dv\right)\right]^{-1} > 0.$$

where the last inequality follows from the properties of the functions $\alpha(\cdot)$ and $\eta(\cdot)$. ■

Lemma D2. The optimal quality q_v of any good $v \in \mathbb{V}$ can be written as follows:

$$q_v = \max\left\{\left[\frac{e^{\eta(0)}\alpha(0)}{e^{\eta(v)}\alpha(v)}\right]^{\frac{1}{\eta(v)-1}} q_0^{\frac{\eta(0)-1}{\eta(v)-1}}, 1\right\}; \tag{31}$$

Proof. Recall that $q_v = 1$, $\forall v \notin \mathbb{L}$. For all other goods, (22) in conjunction with (20) yield:

$$\eta(v) + \ln\alpha(v) + [\eta(v)-1]\ln q_v = \eta(0) + \ln\alpha(0) + [\eta(0)-1]\ln q_0,\ \forall v \in \mathbb{L}.$$

Isolating $[\eta(v)-1]\ln q_v$, and applying exponentials to both sides gives:

$$(q_v)^{\eta(v)-1} = \frac{e^{\eta(0)}}{e^{\eta(v)}}\frac{\alpha(0)}{\alpha(v)}(q_0)^{\eta(0)-1},\ \forall v \in \mathbb{L}.$$

Finally, raising both sides to the power $[\eta(v)-1]^{-1}$, and considering Lemma 1, (31) obtains. ■

Lemma D3. If $\tilde{v}(\kappa) < 1$, then $q_{\tilde{v}(\kappa)} = 1$.

Proof. By definition of $\mathbb{L}$, $\lambda_{\tilde{v}(\kappa)} = 0$. Thus, the condition (22) applied on $\tilde{v}(\kappa)$ yields:

$$\eta\left(\tilde{v}(\kappa)\right) + \ln\left[\alpha\left(\tilde{v}(\kappa)\right)/w\right] - \ln\kappa + \ln Q = -\left[\eta\left(\tilde{v}(\kappa)\right) - 1\right]\ln q_{\tilde{v}(\kappa)} \tag{32}$$

[32] For the rest of the proof, we will assume that the envelope function $\alpha(v)$ is differentiable at all points. It must be straightforward to observe, though, that the function $\alpha(v)$ is strictly increasing in v, since both $a(v)$ and $a^*(v)$ are strictly increasing in v, and that this monotonicity is sufficient to ensure monotonicity of $\widetilde{v}(\kappa)$, which is the important feature of $\widetilde{v}(\kappa)$ that we require in our model.

Suppose now that $q_{\tilde{v}(\kappa)} > 1$, and take some $\varepsilon \in (0, 1 - \tilde{v}(\kappa)]$. Then, since $v = \tilde{v}(\kappa) + \varepsilon \notin \mathbb{L}$, it must be the case that:

$$\eta\left(\tilde{v}(\kappa) + \varepsilon\right) + \ln\left[\alpha\left(\tilde{v}(\kappa) + \varepsilon\right)/w\right] - \ln\kappa + \ln Q = \lambda_{\tilde{v}(\kappa)+\varepsilon}. \tag{33}$$

Then, by continuity of $\eta(\cdot)$ and $\alpha(\cdot)$, and using the result in (32), we must have:

$$\lim_{\varepsilon\to 0}\left\{\eta\left(\tilde{v}(\kappa) + \varepsilon\right) + \ln\left[\alpha\left(\tilde{v}(\kappa) + \varepsilon\right)/w\right] - \ln\kappa + \ln Q\right\} = -\left[\eta\left(\tilde{v}(\kappa)\right) - 1\right]\ln q_{\tilde{v}(\kappa)} < 0.$$

Hence, $q_{\tilde{v}(\kappa)} > 1$ cannot possibly hold when $\tilde{v}(\kappa) < 1$ as it would imply that $\lambda_{\tilde{v}(\kappa)+\varepsilon} < 0$ in (33) for $\varepsilon \to 0$, violating (20). ■

Proof of $\partial\vartheta(m)/\partial w \leq 0$.

Suppose first that $\tilde{v} < m$. Then, $\mathbb{L} \subset [0, m)$. Differentiating (22) with respect to w yields:

$$\frac{\eta(v) - 1}{q_v}\frac{\partial q_v}{\partial w} + \frac{1}{Q}\frac{\partial Q}{\partial w} = 0, \ \forall v \in \mathbb{L}. \tag{34}$$

Furthermore, from (31) it follows that:

$$\frac{\partial q_v}{\partial w} = \frac{\eta(0) - 1}{\eta(v) - 1}\frac{q_v}{q_0}\frac{\partial q_0}{\partial w}, \ \forall v \in \mathbb{L}. \tag{35}$$

Since $\partial Q/\partial w = \int_0^{\tilde{v}} (\partial q_z/\partial w)\, dz$, combining (34) and (35) yields:

$$\left(1 - \tilde{v} + \int_0^{\tilde{v}} \frac{\eta(z)}{\eta(z) - 1} q_z dz\right)\frac{\eta(0) - 1}{q_0}\frac{1}{Q}\frac{\partial q_0}{\partial w} = 0 \quad\Rightarrow\quad \frac{\partial q_0}{\partial w} = 0,$$

Therefore, using again (35), $\partial q_v/\partial w = 0$ for all $v \in [0, \tilde{v}]$ obtains. In addition, because of Lemma 1, it must thus be the case that $\partial q_v/\partial w = 0$ holds as well for all $v \in (\tilde{v}, 1]$. Finally, recalling (6) it then follows that $\partial\beta_v/\partial w = 0$ for all $v \in \mathbb{V}$, which in turn implies that $\partial\vartheta(m)/\partial w = 0$.

Suppose now that $\tilde{v} \geq m$. Differentiating (22) with respect to w now yields:

$$\frac{\eta(v) - 1}{q_v}\frac{\partial q_v}{\partial w} + \frac{1}{Q}\frac{\partial Q}{\partial w} = \begin{cases} 0, & \forall v \in [0, m) \\ 1/w, & \forall v \in [m, \tilde{v}] \end{cases} \tag{36}$$

From (36) it follows that a necessary condition for $\partial\vartheta(m)/\partial w > 0$ to hold is that $\partial Q/\partial w < 0$.[33] However, (36) means that if $\partial Q/\partial w < 0$, then $\partial q_v/\partial w > 0$ should hold for all $v \in [m, \tilde{v}]$. If $\tilde{v} = 1$, it must be straightforward to observe that $\partial Q/\partial w < 0$ cannot thus hold. Alternatively, if

[33] Otherwise, if $\partial Q/\partial w \geq 0$, (36) would imply that $\partial q_v/\partial w \leq 0$ for all $v \in [0, m)$. Recalling (6), it is then straightforward to observe that $\partial Q/\partial w \geq 0$ would mean $\partial\beta_v/\partial w \leq 0$ for all $v \in [0, m)$, which in turn leads to $\partial\vartheta(m)/\partial m \leq 0$.

$\tilde{v} < 1$, then $\partial Q/\partial w < 0$ would require that $\partial q_v/\partial w < 0$ prevails for some $v \in (\tilde{v}, 1]$ which is not feasible either since it would lead to violating the constraint $q_v \leq 1$. As a result, $\partial Q/\partial w \geq 0$ must hold, which in turn implies $\partial\vartheta(m)/\partial w \leq 0$. ■

Proof of $\partial\vartheta^*(m)/\partial w < 0$.

Suppose first that $\tilde{v}^* < m$. Then, $\mathbb{L}^* \subset [0, m)$. Differentiating (22) – adjusted for representing an individual from F – with respect to w yields:

$$\frac{\eta(v) - 1}{q_v^*}\frac{\partial q_v^*}{\partial w} + \frac{1}{Q^*}\frac{\partial Q^*}{\partial w} = -\frac{1}{w},\ \forall v \in \mathbb{L}^*. \tag{37}$$

In addition, from (31) it follows that:

$$\frac{\partial q_v^*}{\partial w} = \frac{\eta(0) - 1}{\eta(v) - 1}\frac{q_v^*}{q_0^*}\frac{\partial q_0^*}{\partial w},\ \forall v \in \mathbb{L}^*. \tag{38}$$

Combining (37) and (38) leads to:

$$\left(1 - \tilde{v}^* + \int_0^{\tilde{v}^*} \frac{\eta(z)}{\eta(z) - 1} q_z dz\right) \frac{\eta(0) - 1}{q_0^*}\frac{1}{Q^*}\frac{\partial q_0^*}{\partial w} = -\frac{1}{w} \quad \Rightarrow \quad \frac{\partial q_0^*}{\partial w} < 0.$$

Hence, using again (38), $\partial q_v^*/\partial w < 0$ for all $v \in [0, \tilde{v}^*]$ obtains, which in turn implies $\partial Q^*/\partial w < 0$. Next, since for all $v \geq \tilde{v}^*$ the constraint $q_v^* \geq 1$ is binding, it must be the case that $\partial q_v^*/\partial w \geq 0$, $\forall v \in (\tilde{v}^*, 1]$. As a result, because of (6), $\partial\beta_v^*/\partial w > 0$ for all $v \in [m, 1]$ follows, which in turn implies $\partial\vartheta^*(m)/\partial w < 0$.

Suppose now $\tilde{v}^* \geq m$. Differentiating (22) with respect to w now yields:

$$\frac{\eta(v) - 1}{q_v^*}\frac{\partial q_v^*}{\partial w} + \frac{1}{Q^*}\frac{\partial Q^*}{\partial w} = \begin{cases} -1/w, & \forall v \in [0, m) \\ 0, & \forall v \in [m, \tilde{v}^*] \end{cases} \tag{39}$$

Suppose $\partial Q^*/\partial w \geq 0$. From (39) it follows that $\partial q_v^*/\partial w < 0$ for all $v \in [0, \tilde{v}^*)$. Furthermore, Lemma 1 then implies that $\partial q_v^*/\partial w \leq 0$ for all $v \in [\tilde{v}^*, 1]$; as a result, $\partial Q^*/\partial w < 0$ must necessarily hold. Now, notice that if $\partial Q^*/\partial w < 0$, then (39) implies $\partial q_v^*/\partial w > 0$ for all $v \in [m, \tilde{v}^*]$. Moreover, in case $\tilde{v}^* < 1$, since $\forall v \in (\tilde{v}^*, 1]$ the constraint $q_v^* \geq 1$ is binding, $\partial q_v^*/\partial w \geq 0$ must necessarily hold for all $v \in (\tilde{v}^*, 1]$. As a result, if $\partial Q^*/\partial w < 0$, then $\partial\beta_v^*/\partial w > 0$ for all $v \in [m, 1]$, which in turn leads to $\partial\vartheta^*(m)/\partial w < 0$. ■

D.2 A Two-Good Simplified Model

This model is a simplified version of Jaimovich and Merella (2010).

Consider a two-good economy. Each good $v = \{0, 1\}$ is potentially producible in two qualities: a baseline quality, conveniently normalised to one ($q_{0l} = q_{1l} = 1$); a refined quality, denoted by $q_{vh} > 1$ for each v. Commodity prices are denoted by p_{vi} for each v, with $i = \{l, h\}$ and $p_{vl} < p_{vh}$. The representative consumer is endowed with w units of resources, fully available for spending. The budget constraint therefore reads:

$$p_{0l}x_{0l} + p_{0h}x_{0h} + p_{1l}x_{1l} + p_{1h}x_{1h} = w$$

Consumer preferences are represented by the function:

$$U = \ln\left(x_{0l} + [x_{0h}]^{q_{0h}}\right) + \ln\left(x_{1l} + [x_{1h}]^{q_{1h}}\right)$$

The representative consumer solves:

$$\begin{array}{rl} \max_{\{x_{vi}\}} & \sum_{v=\{0,1\}} \ln\left(\sum_{i=\{l,h\}} (x_{vi})^{q_{vi}}\right) \\ s.t. & \sum_{v=\{0,1\}} \sum_{i=\{l,h\}} p_{vi}x_{vi} = w \end{array}$$

The additive specification of the utility function conveniently allows to solve the problem in two steps. First, for a given budget share devoted to spending on good v, denoted by β_v, the consumer chooses which quality to consume by solving:

$$\begin{array}{rl} \max_{\{x_{vi}\}} & \ln\left(\sum_{i=\{l,h\}} (x_{vi})^{q_{vi}}\right) \\ s.t. & \sum_{i=\{l,h\}} p_{vi}x_{vi} = \beta_v w \end{array}$$

To solve this problem, note that the utility function is convex in $\{x_{vi}\}$. The problem thus delivers a corner solution, i.e. $x_{vi} = \beta_v w/p_{vi}$ and $x_{vj} = 0$, with $j \neq i$. The solution is found by comparing the utility yielded by consuming either quality. The consumer chooses to consume quality q_{vl} if:

$$\beta_v w/p_{vl} \geq (\beta_v w/p_{vh})^{q_{vh}}$$

and quality q_{vh} otherwise. Hence:

$$\begin{array}{ll} x_{vl} = \beta_v w/p_{vl} \text{ and } x_{vh} = 0 & \text{if } \beta_v w \leq (p_{vh})^{(p_{vh})^{\frac{q_{vh}}{q_{vh}-1}}} (p_{vl})^{\frac{1}{1-q_{vh}}} \\ x_{vl} = 0 \text{ and } x_{vh} = \beta_v w/p_{vh} & \text{if } \beta_v w > (p_{vh})^{\frac{q_{vh}}{q_{vh}-1}} (p_{vl})^{\frac{1}{1-q_{vh}}} \end{array}$$

Second, given the optimal quality, denoted by q_v (and relevant price, p_v), the consumer chooses the fractions of resources to devote to the two goods by solving:

$$\begin{array}{rl} \max_{\{\beta_v\}} & \sum_{v=\{0,1\}} q_v \ln\left(\frac{\beta_v w}{p_{vi}}\right) \\ s.t. & \sum_{v=\{0,1\}} \beta_v = 1 \end{array}$$

To solve this problem, we may write the Lagrangian as:

$$\mathcal{L} = \sum_{v=0,1} q_v \ln\left(\frac{\beta_v w}{p_{vi}}\right) + \mu\left(1 - \sum_{v=0,1} \beta_v\right)$$

which delivers the first-order conditions:

$$\begin{array}{l} \frac{q_v}{\beta_v} = \mu, \text{ with } v = 0,1 \\ \sum_{v=\{0,1\}} \beta_v = 1 \end{array}$$

Combining these conditions yields:

$$\begin{aligned} q_0/\left(1-\beta_1\right) &= q_1/\beta_1 \\ \beta_1 q_0 &= q_1 - \beta_1 q_1 \\ \beta_1 &= q_1/\left(q_0+q_1\right) \end{aligned}$$

and, similarly:

$$\beta_0 = q_0/\left(q_1+q_2\right)$$

Denoting total quality by $Q = q_1 + q_2$, and replacing these two equations in those that solve the first problem yields:

$$\begin{array}{ll} x_{vl} = w/\left(p_{vl}Q\right) \text{ and } x_{vh} = 0 & \text{if } w \leq \left(p_{vh}\right)^{\frac{q_{vh}}{q_{vh}-1}}\left(p_{vl}\right)^{\frac{1}{1-q_{vh}}} Q \\ x_{vl} = 0 \text{ and } x_{vh} = q_{vh}/\left(p_{vh}Q\right) & \text{if } w > \left(p_{vh}\right)^{\frac{q_{vh}}{q_{vh}-1}}\left(p_{vl}\right)^{\frac{1}{1-q_{vh}}} Q/q_{vh} \end{array}$$

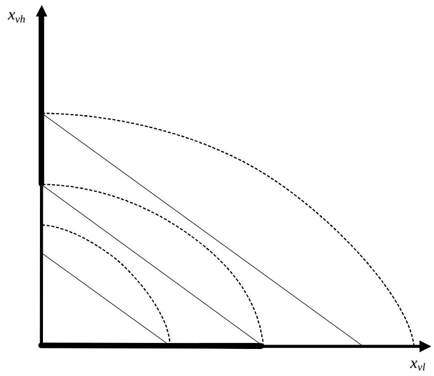
We can thus observe that, for finite prices, each good will eventually upgrade qualitatively, as illustrated by Figure A1.

To understand the effect of quality upgrading on the budget shares, we need to be more specific on the behaviour of prices. To this aim, we let prices be:

$$p_{vi} = \alpha\left(v\right)\left(q_{vi}\right)^{\eta(v)}$$

where $\alpha(v) > 0$ and $\eta(v) > 1$ are good-specific technological parameters (with the latter governing the cost elasticity of quality upgrading). Note that, compared to Jaimovich and

Figure A1: Intra-good wealth expansion path



The quantities consumed of the low- and high-quality goods (x_{vl} and x_{vh}, respectively) are measured on the axes (horizontal and vertical, respectively). The parallel solid lines represent the intra-good budget constraint for three different levels of income w (from left to right, less than, equal to and greater than w_0, respectively). The dotted lines represent indifference curves. The ticker solid segments on the axes represent the wealth expansion path.

Merella (2010), we normalise the value of the aggregate productivity parameter $\kappa = 1$. Then, we have:

$$p_{vl} = \alpha(v)\,;\; p_{vh} = \alpha(v)\,(q_{vh})^{\eta(v)}$$

and:

$$\begin{array}{ll} x_{vl} = w/\left[\alpha(v)\,Q\right] \text{ and } x_{vh} = 0 & \text{if } w \leq \alpha(v)\,(q_{vh})^{\frac{\eta(v)q_{vh}}{q_{vh}-1}}\,Q \\ x_{vl} = 0 \text{ and } x_{vh} = w/\left[\alpha(v)\,(q_{vh})^{\eta(v)-1}\,Q\right] & \text{if } w > \alpha(v)\,(q_{vh})^{\frac{\eta(v)q_{vh}}{q_{vh}-1}}\,Q \end{array}$$

Finally, denote:

$$w_0 = \alpha(0)\,(q_{0h})^{\frac{\eta(0)q_{0h}}{q_{0h}-1}}\,Q; \quad w_1 = \alpha(1)\,(q_{1h})^{\frac{\eta(1)q_{1h}}{q_{1h}-1}}\,Q$$

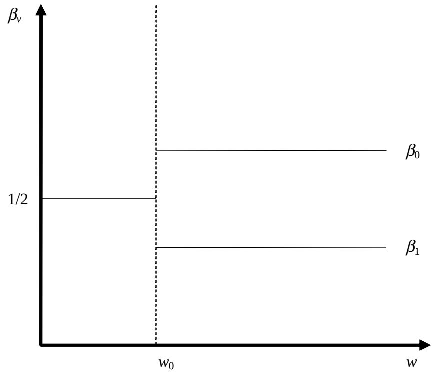
In line with the Jaimovich and Merella (2010) benchmark model, consider first the case: $\alpha(0) \leq \alpha(1)$ and $\eta(0) < \eta(1)$. In addition, assume $\eta(1) \to \infty$. In this case, $w_0 < w_1 = \infty$, since:

$$\alpha(0)\,(q_{0h})^{\frac{\eta(0)q_{0h}}{q_{0h}-1}} \leq \alpha(1)\,(q_{1h})^{\frac{\eta(1)q_{1h}}{q_{1h}-1}} = \infty$$

The budget shares are $\beta_0 = \beta_1 = 1/2$ as long as $w \leq w_0$, beyond where β_0 rises above $1/2$ as $w > w_0$, as illustrated in Figure A2.

Other possible dynamics can be illustrated by changing the above assumptions.

Figure A2: Inter-good budget allocations (Engel curves)



Income is measured on the horizontal axis; the budget shares spent on consumption of the two differentiated goods (β_v, with v = 0,1) are measured on the vertical axis. The solid lines depicting the optimal budget shares (which diverge when income w becomes greater than w_0) represent the Engel curves in a budget share form.

Consider, now, the case: $\alpha(0) \leq \alpha(1)$ and $\eta(0) < \eta(1)$, assuming $\eta(1)$ is finite. In this case, the budget shares are $\beta_0 = \beta_1 = 1/2$ as long as $w \leq w_0$, beyond where β_0 rises (again) above $1/2$ as $w > w_0$. This is eventually followed by an increase in β_1 as $w > w_1$. Depending on the relative value of the high-quality levels, the catching up may be partial ($q_{0h} > q_{1h}$), full ($q_{0h} = q_{1h}$), or β_1 may even overtake β_0 ($q_{0h} < q_{1h}$)

Finally, consider the case: $\alpha(0) > \alpha(1)$ and $\eta(0) < \eta(1) < \infty$. If $\alpha(1)$ is small enough relative to $\alpha(0)$, then it may be that:

$$w_0 = \alpha(0)(q_{0h})^{\frac{\eta(0)q_{0h}}{q_{0h}-1}} > \alpha(1)(q_{1h})^{\frac{\eta(1)q_{1h}}{q_{1h}-1}} = w_1$$

Mirroring the previous case, the budget shares are $\beta_0 = \beta_1 = 1/2$ as long as $w \leq w_1$, then β_1 rises above $1/2$ as $w > w_1$. This is subsequently followed by an increase in β_0 as $w > w_1$. Once again, depending on the relative value of the high-quality levels, the catching up may be partial ($q_{0h} < q_{1h}$), full ($q_{0h} = q_{1h}$), or β_0 may even rise above β_1.

D.3 Unit prices at 1-digit level disaggregation

In Table A1 we group all the SITC-4 sectors/goods into their corresponding 1-digit sector. Therein we report the 1-digit level average values of the interdecile unit price ratios and the coefficients of variation of unit prices, both calculated at the 4-digit level of disaggregation. With the exception of sector 2, Table A1 seems to point to the common perception that the quality ladders of primary goods tend to be shorter than those of manufacturing products (i.e. sectors 5 to 8).

Table A1: Averages at 1-digit level of disaggregation

SITC-1 Sector	Number of products in SITC-4 classif.	Average of max-to-min unit price ratios	Average of coeff. of variation of unit prices
0 - Food and live animals	93	4.46	0.711
1 - Beverages and tobacco	11	4.86	0.656
2 - Crude materials, inedible, except fuels	101	8.80	1.084
3 - Mineral fuels, lubricants and rel. materials	20	3.21	0.876
4 - Animal and vegetable oils, fats and waxes	18	2.67	0.526
5 - Chemicals and related products	91	5.67	1.024
6 - Manufactured goods classified chiefly by material	175	5.34	0.952
7 - Machinery and transport equipment	157	7.54	0.919
8 - Miscellaneous manufactured articles	78	9.94	0.887
ALL GOODS	**744**	**6.38**	**0.927**